```
(* 2016.10.22 This is to show 3dPlot in the same scale for all the axes *)

(* g1, g2 for Cobb-Douglas, pla, pl2 for planes.*)
(* Remove["Global`*"] *)


Text[Style["Cobb-Douglas Utility Function with Two Goods: x^(1/3)y^(2/3)", "Title"]]
g1 = Plot3D[(x^(1 / 3)) * (y^(2 / 3)), {x, 0, 5}, {y, 0, 5},
  AxesLabel → {"x", "y", "z"}, LabelStyle → Directive[Bold, Large], ImageSize → Large,
  FaceGrids -> All, BoundaryStyle -> Directive[Black, Thickness[0.015]],
  BoxRatios -> Automatic, PlotRange -> {{0, 5}, {0, 5}, {0, 5.7}}]

Style["By the way, this is how Cobb-Douglas Function, x^(2/3)y^(1/3), looks like", "Title"]
Plot3D[(x^(2 / 3)) * (y^(1 / 3)), {x, 0, 5}, {y, 0, 5},
 AxesLabel → {"x", "y", "z"}, ImageSize → Large, FaceGrids -> All,
 BoundaryStyle -> Directive[Black, Thickness[0.015]],
 BoxRatios -> Automatic, PlotRange -> {{0, 5}, {0, 5}, {0, 5.7}}]


Style[
 "Here we use the Cobb-Douglas Function: x^(1/3)y^(2/3). Let us see it in a full scale.",
 "Title"]
Show[g1, ImageSize → Full]

(* By the way, we can derive indifference curves *)
(* Inddiference curve at z = 2 *)

Text[Style["Before thinking about partial
    differnatiation, let us consider Indifference Curve", Blue, 24]]
Text[Style["Let us cut z = x^(1/3)y^(2/3) at z = 1.5", Black, 24]]

pl1 = ContourPlot3D[z == 1.5, {x, 0, 5},
  {y, 0, 5}, {z, 0, 5.7}, AxesLabel → {"x", "y", "z"},
  LabelStyle → Directive[Bold, Large], ImageSize → Large, ContourStyle → Blue]
Show[g1, pl1]

Text[Style["The Contour made by z=1.5 plane gives us an indifference curve.", 24]]
ContourPlot[(x^(1 / 3)) * (y^(2 / 3)) == 1.5,
 {x, 0, 5}, {y, 0, 5}, AxesLabel → {"x", "y"}]

Text[Style["By cutting with different value 'z's, we can have many
    idifference curves representing different utility level.", 24]]
ContourPlot[(x^(1 / 3)) * (y^(2 / 3)), {x, 0, 5}, {y, 0, 5},
 AxesLabel → {"x", "y"}, PlotLegends → Automatic]

(* try to draw y = 1 plane *)
(*s1={{0,1,0},{0,1,5.7},{5,1,0},{5,1,5.7}}
  Show [Graphics3D[Polygon[s1], AxesLabel→{"x","y","z"}]]*)
Text[
```

```
 Style["But our main purpose now is to understand partial derivatives!", "Title"]]
Text[ Style["Recall that a derivative in 2D is a slope that approximates
    the original curve at a specific point. How about in 3D?", 24]]
Text[ Style["Let us consider x^(1/3) y^(2/3) at (x, y) = (2, 1).", 24]]
label1 = Graphics3D[Text[Style["(2, 1, 2^(1/3) 1^(2/3))", Blue, 28], {2, 1, 1.6}]];


po1 = ListPointPlot3D[{{2, 1, N[2^(1/3) × 1^(2/3)]}},
   AxesLabel → {"x", "y", "z"}, BoxRatios → Automatic, PlotStyle → PointSize[0.03]];

 Show[g1, po1, label1]

Text[Style["Magniy it around (x, y, z) = (2, 1, 2^(1/3) 1^(2/3))", 24]]
g11 = Show[Plot3D[{(x^(1/3)) * (y^(2/3))},
   {x, 1.5, 2.5}, {y, 0.5, 1.5}, AxesLabel → {"x", "y", "z"},
   LabelStyle → Directive[Bold, Large], ImageSize → Full, FaceGrids -> All,
   BoundaryStyle -> Directive[Black, Thickness[0.02]], BoxRatios -> Automatic], po1]

Text[
 Style["We can guess that a plane is a strong candidate for approximating this curved
    surface at (x, y, z) = (2, 1, 2^(1/3)*1^(2/3))", 24]]

Text[Style["Let us confirm our guess.", "Title"]]

Text[Style["Fix y at some value, then look at
    our 3D graphic from x axis. The 3D becones like a 2D.", 24]]
Text[Style["Here we fix y at 1.", 24]]

pl2 = ContourPlot3D[y == 1, {x, 0, 5},
  {y, 0, 5}, {z, 0, 5.7}, AxesLabel → {"x", "y", "z"},
  LabelStyle → Directive[Large, Bold], ImageSize → Large, ContourStyle → Black]


Show[g1, pl2]


g2 = Plot[{(x^(1/3)) * (1^(2/3))}, {x, 0, 5},
  AxesLabel → {"x", "z"}, AspectRatio → Automatic, ImageSize → Large,
  LabelStyle → Directive[Bold, Large], PlotStyle → {Black}]


Text[Style["If we change the value of y, the pseudo
    2D graphs change its shape a bit.  Here we fix y at 3.", 24]]
pl3 = ContourPlot3D[y == 3, {x, 0, 5}, {y, 0, 5}, {z, 0, 5.7},
  AxesLabel → {"x", "y", "z"}, ImageSize → Large,
  ContourStyle → Red, LabelStyle → Directive[Large, Bold]]

Show[g1, pl3]
(*N[3^(-(1/3))]*)
```

```
Text[Style["The contour is another 2D graphic", 24]]
g3 = Plot[(x^(1 / 3)) * (3^(2 / 3)), {x, 0, 5},
  AxesLabel → {"x", "z"}, AspectRatio → Automatic, ImageSize → Large,
  PlotStyle → {Red}, LabelStyle → Directive[Bold, Large]]


Text[Style["We can see the differnces by the cutting values of y", 24]]

Plot[{(x^(1 / 3)) * (1^(2 / 3)), (x^(1 / 3)) * (3^(2 / 3))}, {x, 0, 5},
 AspectRatio → Automatic, ImageSize → Large, PlotRange -> {{0, 5}, {0, 3.6}},
 PlotStyle → {Black, Red}, PlotLegends -> {"at y = 1", "at y =3"}]


Text[Style["Now we focus on the 2D with y =1.", Black, "Title"]]
Text[Style["Differentiate (x^(1/3))*(1^(2/3)) with respect x,", 24]]

D[(x^(1 / 3)) * (1^(2 / 3)), x]

Text[
 Style["Differentiate x^(1/3))*(y^(2/3) with respect x. Here, we are doing partial
    differnetiation. Please accept the result at this stage.", 24]]
D[(x^(1 / 3)) * (y^(2 / 3)), x]


Text[Style["Derivative (= scalar) of 2D at x = 2", 24]]
s1 = N[D[(x^(1 / 3)) * (1^(2 / 3)), x] /. x → 2]

Text[Style["Partial derivative of 3D at (x, y) = (2, 1)", 24]]
N[D[(x^(1 / 3)) * (y^(2 / 3)), x] /. {x → 2, y → 1}]


Text[Style[
  "In short, partial derivative in 3D is a `slope' of 2D after fixing y (or x)", 24]]
(* The vale of (x^(1/3))*(1^(2/3)) at x = 2 *)
v1 = N[(x^(1 / 3)) * (1^(2 / 3)) /. x → 2];
(* by putting cumma, the output is not shown *)
(* b1: y intercept: set a line going through (2,v1) *)
b1 = v1 - s1 * 2;

Text[
 Style["Partial derivative with respect to x at (x, y, z)=(2, 1, 2^(1/3))*(1^(2/3))
    is a slope of (x^(1/3))*(1^(2/3)) at x = 2", 24]]
Text[Style["With the 'scalar value' of slope and the information
    that the slope is evaluated at (x, z) = (2,
    2^(1/3))*(1^(2/3)), we can derive a tangent line", Blue, 24]]

Text[Style[
  "The derived tangent line is z = 0.8399473665965822 + 0.20998684164914552*x", 24]]

y1 = Plot[{ b1 + s1 * x}, {x, 0, 5}, ImageSize → Large,
```

```
  AxesOrigin → {0, 0}, PlotStyle → {Blue}, AxesLabel → {"x", "z"},
  LabelStyle → Directive[Large, Bold], AspectRatio → Automatic]
Show[g2, y1, ImageSize → Large]


Text[Style["As usual, tangent lines derived
    from derivatives are Very good approximates of curves", 24]]
Text[Style["Let us magnify the 2D graph and the tangebt line around x = 2", 24]]


Plot[{{b1 + s1 * x}, {(x^(1 / 3)) * (1^(2 / 3))}}, {x, 1.8, 2.2},
 PlotStyle → {Blue, Black}, ImageSize → Full, AxesLabel → {"x", "z"},
 LabelStyle → Directive[Bold, Large], AspectRatio → Automatic]


Text[Style[
  "Let us have a look at partial derivative with respect to x at (2,1) on 3D", 24]]
b1 + s1 * 2;
N[2^(1 / 3)]; (*This is to confirm the z value a (x,y) = (2,1) *)


l1 = Graphics3D[
   {Blue, Thick, Line[{{0, 1, b1}, {2, 1, b1 + s1 * 2}, {4.8, 1, b1 + s1 * 4.8}}]}];

Show[g1, l1, po1]
(*Graphics3D[Arrow{{2,1,N[2^(1/3) 1^(2/3)]},{2.5 ,1,N[2^(1/3)+(s1*0.5)]}}] *)



Text[Style["We can do the similar procedure by fixing x at 2", 24]]
pl4 = ContourPlot3D[x == 2, {x, 0, 5},
  {y, 0, 5}, {z, 0, 5.7}, AxesLabel → {"x", "y", "z"},
  ImageSize → Large, ContourStyle → Green, PlotLabel → "x = 2"]

Show[g1, pl4]

Text[Style["The contour is a 2D graphic", 24]]

g4 = Plot[{(2^(1 / 3)) * (y^(2 / 3))}, {y, 0, 5},
  AxesLabel → {"y", "z"}, LabelStyle → Directive[Large, Bold],
  AspectRatio → Automatic, ImageSize → Large, PlotStyle → Green]


Text[Style["Differentiate (2^(1/3))*(y^(2/3)) with respect y,", 24]]

D[(2^(1 / 3)) * (y^(2 / 3)), y]

Text[Style["Differentiate (x^(1/3))*(y^(2/3)) with
    respect y. Here, we are doing partial differnetiation.", 24]]
D[(x^(1 / 3)) * (y^(2 / 3)), y]
```

```
Text[Style["Derivative of 2D at y = 1", 24]]
s2 = N[D[(2^(1 / 3)) (y^(2 / 3)), y] /. y → 1]

Text[Style["Partial derivative of 3D at (x, y) = (2, 1)", 24]]
N[D[(x^(1 / 3)) * (y^(2 / 3)), y] /. {x → 2, y → 1}]
N[(2 / 3) * 2^(1 / 3)];

v2 = N[(2^(1 / 3)) * (y^(2 / 3)) /. y → 1];

(* Derive y intercept *)
(* Solve[ v2 == (s2*1)+b1,b1];*)
b2 = v2 - s2 * 1;

Text[
 Style["Partial derivative with respect to y at (x, y, z)=(2, 1, 2^(1/3))*(1^(2/3))
    is a slope of (2^(1/3))*(y^(2/3)) at y = 1", 24]]
Text[Style["With the 'scalar value' of slope and the information
    that the slope is evaluated at (y, z) = (1,
    2^(1/3))*(1^(2/3)), we can derive a tangent line", Blue, 24]]
Text[Style["The derived tangent line is z = 0.4199736832982911
    + 0.8399473665965821*y", 24]]

y2 = Plot[{b2 + s2 * y}, {y, 0, 5}, AxesOrigin → {0, 0}, AxesLabel → {"y", "z"},
  LabelStyle → Directive[Large, Bold], AspectRatio → Automatic]
Show[g4, y2, ImageSize → Large]

Text[Style["Partial derivative with respect to y at (2, 1, 2^{1/3}1^{2/3}) on 3D", 24]]


l2 = Graphics3D[
   {Blue, Thick, Line[{{2, 0, b2}, {2, 1, b2 + s2 * 1}, {2, 4.8, b2 + s2 * 4.8}}]}];


Show[g1, l2, po1]
(*Graphics3D[Arrow{{2,1,N[2^(1/3) 1^(2/3)]},{2.5 ,1,N[2^(1/3)+(s1*0.5)]}}] *)

Text[Style["If we combine the two tangent lines on 3D", 24]]
Show[g1, l1, l2, po1]
(*Graphics3D[Arrow{{2,1,N[2^(1/3) 1^(2/3)]},{2.5 ,1,N[2^(1/3)+(s1*0.5)]}}] *)
(*
Show[g1,Graphics3D[ Arrow[{{2,1,N[2^(1/3)*1^(2/3)]},
    {2,2,N[2^(1/3)*1^(2/3)+(0.8399473665965821)]}}]],Graphics3D[
  Arrow[{{2,1,N[2^(1/3)*1^(2/3)]},{3,1,2^(1/3)+(0.20998684164914552)}}]],
 Graphics3D[ Arrow[{{2,1,N[2^(1/3)*1^(2/3)]},{2,1,(2^(1/3))-1}}]]]
*)

Text[Style["The two tangent lines in 3D give us a
    plane: A Tangent Plane on (x, y, z) = (2, 1, 2^{1/3}1^{2/3})", Red, 24]]
```

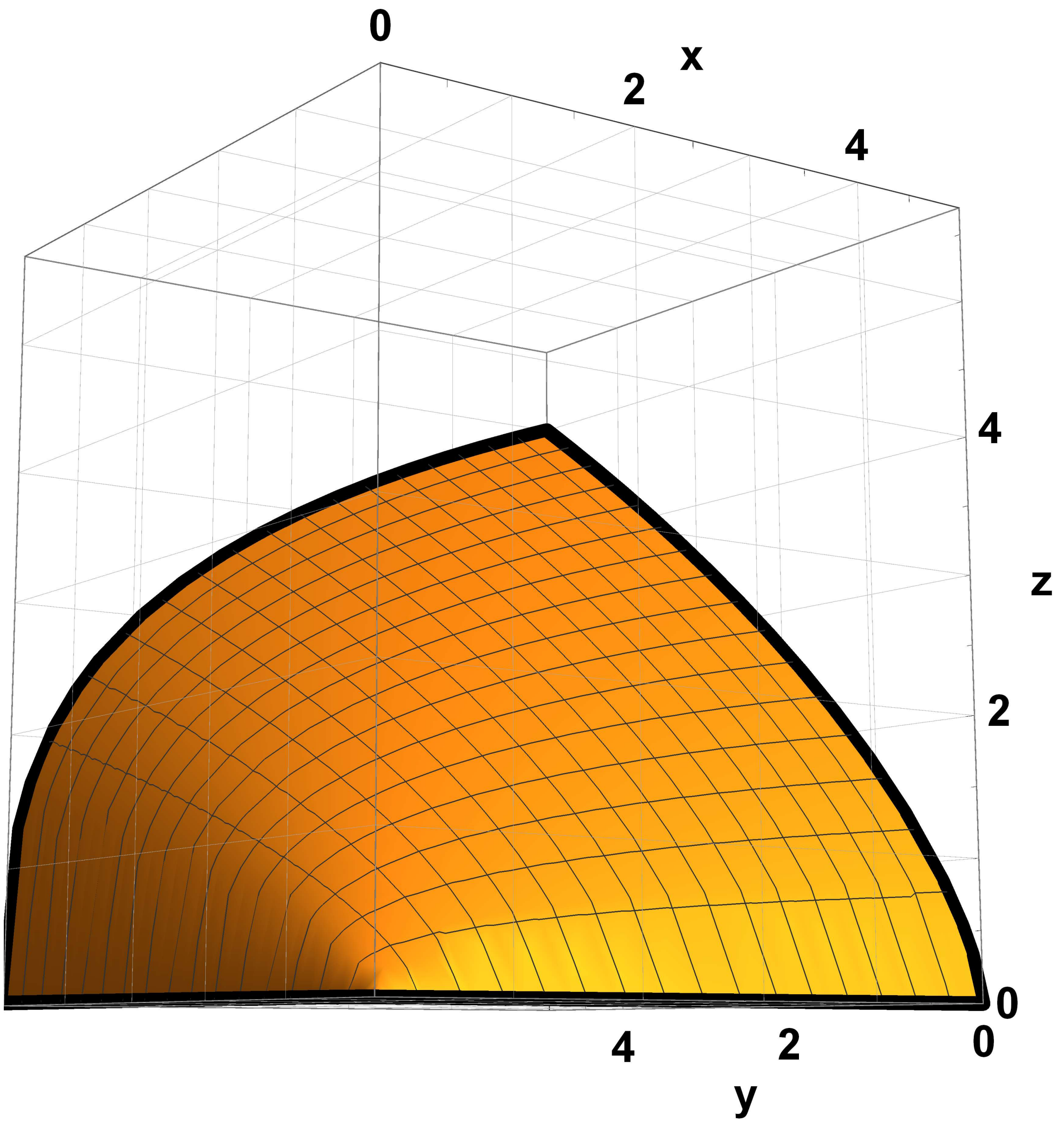
```
d[x_, y_] = ((1 / 3) (2^(- (2 / 3)))) * (x) + ((2 / 3) * (2)^(1 / 3)) * (y)
plane1 = Plot3D[d[x, y], {x, 1.5, 2.5}, {y, 0.5, 1.5},
        AxesLabel -> {"x", "y", "z"}, ImageSize → Large,
   PlotStyle → Red, LabelStyle → Directive[Large, Bold], BoxRatios → Automatic];

Show[plane1, po1, l1, l2]

Text[
 Style["Recall that our goal is to use derivatices (calculus) in economics: linear
    approximation of 'non-linear' relationships", Red, 24]]

(*Text[
 Style["On the tangent Plane at (x, y) = (2,1), let us put a point on 'Non-linear'
    Surface where (x, y) = (2.3, 1.4).",Blue, 24]]*)
Text[Style["Can the tangent Plane at (x, y, z) = (2, 1, 2^(1/3)1^(2/3))
    approxiapiate the curved surface at, for example,
    (x, y, z) = (2.3, 1.4, 2.3^(1/3)1.4^(2/3)) well?", 24]]

po22 = ListPointPlot3D[{{2.3, 1.4, N[2.3^(1 / 3) × 1.4^(2 / 3)]}},
   AxesLabel → {"x", "y", "z"}, BoxRatios → Automatic,
   PlotStyle → PointSize[0.09], LabelStyle → Directive[Large, Bold]];
po11 = ListPointPlot3D[{{2, 1, N[2^(1 / 3) × 1^(2 / 3)]}},
   AxesLabel → {"x", "y", "z"}, BoxRatios → Automatic,
   PlotStyle → PointSize[0.09], LabelStyle → Directive[Large, Bold]];

(* make point size larger to emphasize the approximtion *)
label2 =
  Graphics3D[Text[Style["(2.3, 1.4, 2.3^(1/3)1.4^(2/3))", Blue, 24], {2.3, 1.4, 1.8}]];



Show[plane1, po11, po22, label1, label2]

Text[
 Style["You can see that the point at (x, y, z) = (2.3, 1.4, 2.3^(1/3)1.4^(2/3)) dipped into
    the plane a little bit, but the plane is surely a good approximate.", 24]]

Text[Style["Let us see this approximation by the
    plane on the oroginal 3D surface of Cobb-Douglas function", 24]]
po2 = ListPointPlot3D[{{2.3, 1.4, N[2.3^(1 / 3) × 1.4^(2 / 3)]}},
   AxesLabel → {"x", "y", "z"}, BoxRatios → Automatic, PlotStyle → PointSize[0.03]];

Show[g1, plane1, l1, l2, po1, po2]

Text[Style["Looks like a good approximation, but difficult to see.
    Let us magnify the graph around (x, y, z) = (2, 1, 2^(1/3)1^(2/3))", 24]]
label3 = Graphics3D[Text[Style["(2.3, 1.4, 2.3^(1/3)1.4^(2/3))", Blue, 28],
    {2.3, 1.4, 1.7}]];
```

```
Show[g11, plane1, l1, l2, po1, po2, label3]

Text[Style[
  "With this 3D Graph, we can easily understand so-called 'Total Differentiation'",
  24]]
Text[Style["Let us rewrite our Cobb-Douglas function
    in an abstract way: z = f(x, y) = x^(1/3) y^(2/3)", 24]]

Text[Style["'Total Differentiation': dz = ∂/∂x f(x, y) dx + ∂/∂y f(x, y) dy.", 24]]
Text[
 Style["In plain English, we would like to know how much does the functional value
    z change when both x and y change a little bit", 24]]


Text[
 Style["To understand the total differnatiation on a graph, let us consider fairly
    big changes in x and y: dx = 1.8 = (3.8-2), dy = 1.5 = (2.5 -1)", 24]]


ar1 = Graphics3D[
   {Blue, Thick, Arrow[{{2, 1, N[2^(1/3) * 1^(2/3)]}, {3.8, 1, b1 + s1 * 3.8}}]}];
ar11 = Graphics3D[{Black, Thick,
    Arrow[{{2, 1, N[2^(1/3) * 1^(2/3)]}, {3.8, 1, N[2^(1/3) * 1^(2/3)]}}]}];
ar12 = Graphics3D[{Green, Thick, Arrow[
     {{3.8, 1, N[2^(1/3) * 1^(2/3)]}, {3.8, 1, b1 + s1 * 3.8}}]}];

ar2 = Graphics3D[
   {Blue, Thick, Arrow[{{2, 1, N[2^(1/3) * 1^(2/3)]}, {2, 2.5, b2 + s2 * 2.5}}]}];
ar21 = Graphics3D[{Black, Thick,
    Arrow[{{2, 1, N[2^(1/3) * 1^(2/3)]}, {2, 2.5, N[2^(1/3) * 1^(2/3)]}}]}];
ar22 = Graphics3D[{Yellow, Thick, Arrow[
     {{2, 2.5, N[2^(1/3) * 1^(2/3)]}, {2, 2.5, b2 + s2 * 2.5}}]}];

plane2 = Plot3D[d[x, y], {x, 1, 4}, {y, 0.5, 3},
       AxesLabel -> {"x", "y", "z"}, ImageSize → Large,
   LabelStyle → Directive[Large, Bold], PlotStyle → Red, BoxRatios → Automatic];

po3 = ListPointPlot3D[{{3.8, 2.5, N[3.8^(1/3) × 2.5^(2/3)]}} ,
   LabelStyle → Directive[Large, Bold], AxesLabel → {"x", "y", "z"},
   BoxRatios → Automatic, PlotStyle → PointSize[0.03]];

l3 = Graphics3D[{Red, Thickness[0.01], Line[{{3.8, 2.5, N[2^(1/3) × 1^(2/3)]},
      {3.8, 2.5, N[3.8^(1/3) × 2.5^(2/3)]}}] }];
(*ld1 = Graphics3D[{Thick,Line[{{2,1,[2^(1/3) 1^(2/3)]},
     {3.8,2.5, N[2^(1/3) 1^(2/3)]}}]}]*)
ld1 = Graphics3D[{Dashed, Thick, Line[{{2, 1, N[2^(1/3) × 1^(2/3)]},
      {3.8, 2.5, N[2^(1/3) × 1^(2/3)]}}] }];
```

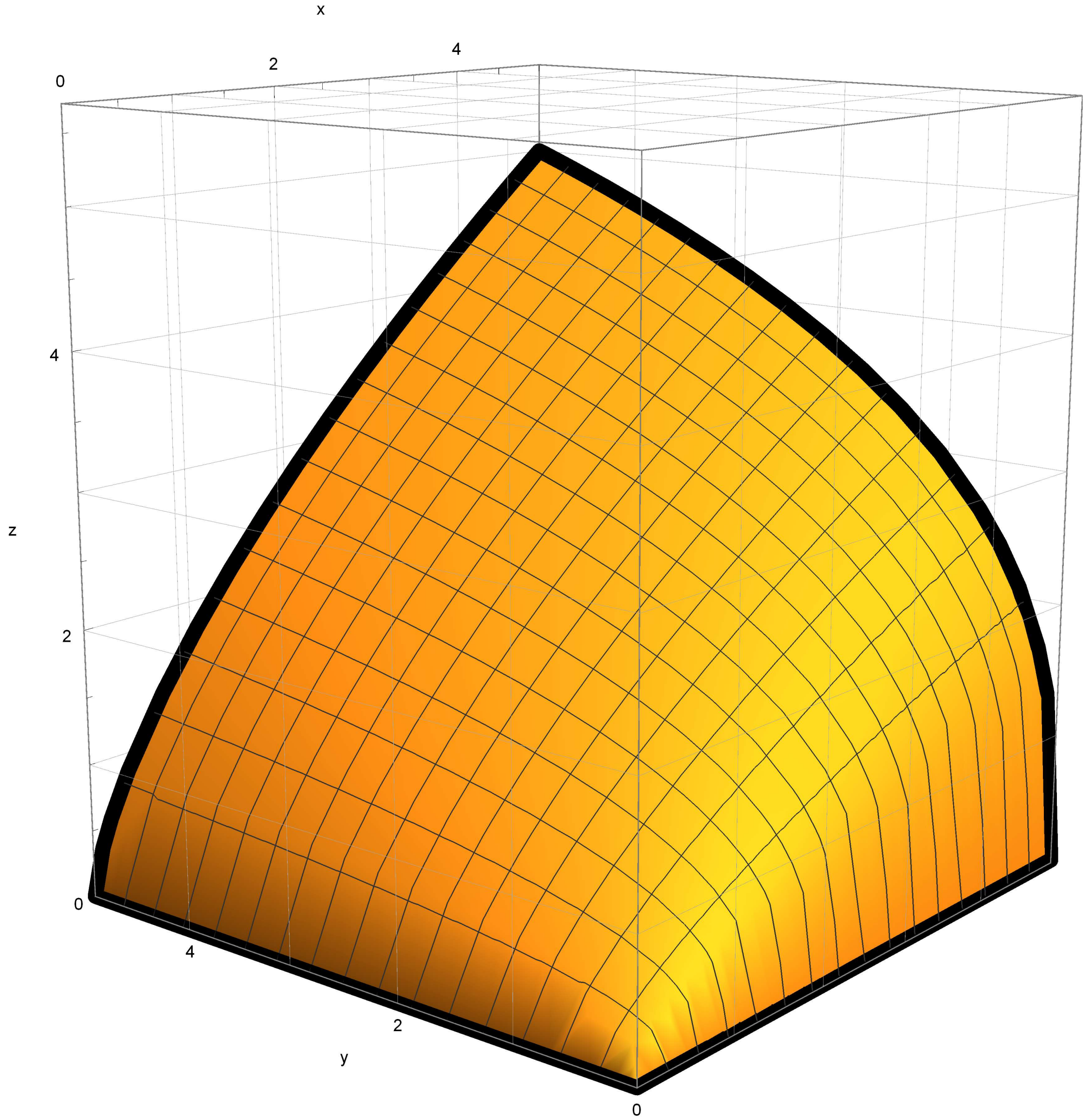
```
label4 =
  Graphics3D[Text[Style["(3.8, 2.5, 3.8^(1/3)2.5^(2/3))", Blue, 28], {3.8, 2.5, 3.5}]];

Text[Style["See the at Graph at first.", 24]]

Show[g1, ar1, ar11, ar12, ar2, ar21, ar22,
 l3, ld1, po1, po3, plane2, label4, ImageSize → Full]


Text[Style["What total differentiation does is:", 24]]



l4 = Graphics[{Red, Thickness[0.03],
    Line[{{2, 0}, {2, N[3.8^(1/3) × 2.5^(2/3)] - N[2^(1/3) * 1^(2/3)]}}]}];
l5 = Graphics[{Green, Thickness[0.03],
    Line[{{1, 0}, {1, (b1 + s1 * 3.8 - N[2^(1/3) * 1^(2/3)])}}]}];
l6 = Graphics[{Yellow, Thickness[0.03],
    Line[{{1, (b1 + s1 * 3.8 - N[2^(1/3) * 1^(2/3)])},
      {1, (b1 + s1 * 3.8 - N[2^(1/3) * 1^(2/3)] + b2 + s2 * 2.5 - N[2^(1/3)])}}]}];
l7 = Graphics[{Black, Thickness[0.01], Dashed,
    Line[{{0.5, (N[3.8^(1/3) × 2.5^(2/3)] + 0.02 - N[2^(1/3) * 1^(2/3)])},
      {2.5, (N[3.8^(1/3) × 2.5^(2/3)] + 0.02 - N[2^(1/3) * 1^(2/3)])}}]}];

Show[l4, l5, l6, l7, ImageSize → Large]

Text[
 Style["The tangent plane is avobe the curve => A little bit overestimation:", 24]]
Text[Style["The length of red line is 3.8^(1/3)2.5^(2/3) - 2^(1/3)1^(2/3) =", 24]]

N[3.8^(1/3) × 2.5^(2/3)] - N[2^(1/3) * 1^(2/3)]

Text[Style["while the sum of green and yellow is", 24]]

N[b1 + s1 * 3.8 - N[2^(1/3) * 1^(2/3)] + b2 + s2 * 2.5 - N[2^(1/3)]]


Text[Style["An alternative and more intuitive interpretation is
    to think a total differntiation as a summation of 2 vectors", 24]]

ar3 = Graphics3D[{Blue, Thickness[0.02], Arrow[{{2, 1, N[2^(1/3) × 1^(2/3)]},
      {3.8, 2.5, (b1 + s1 * 3.8 + b2 + s2 * 2.5 - N[2^(1/3) × 1^(2/3)])}}]}];

ar31 = Graphics3D[{Green, Thick,
    Arrow[{{3.8, 2.5, N[2^(1/3) × 1^(2/3)]}, {3.8, 2.5, b1 + s1 * 3.8}}]}];

ar32 = Graphics3D[{Yellow, Thick, Arrow[{{3.8, 2.5, b1 + s1 * 3.8},
      {3.8, 2.5, b1 + s1 * 3.8 + b2 + s2 * 2.5 - N[2^(1/3) * 1^(2/3)]}}]}];
```

```
Show[ar1, ar11, ar12, ar2, ar21, ar22, ar3, ar31, ar32, ld1, po1, ImageSize → Large]
Text[
 Style["Here the thick blue arrow is the gradient vector: ∇f = (f₁, f₂) at (x, y) =
    (2, 1). It gives us the steppest increase (slope) at (2, 1).", 24]]
```

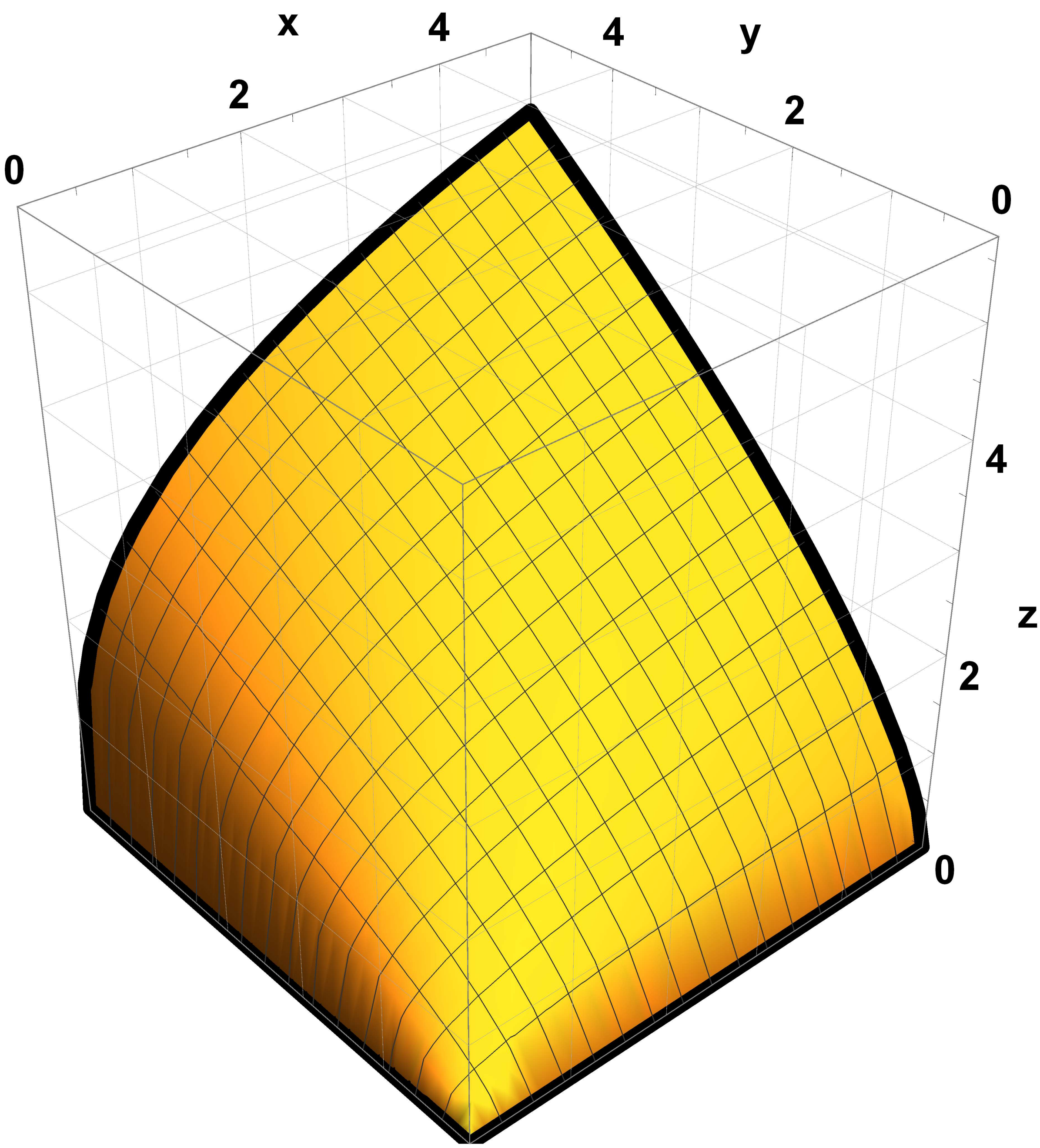
# Cobb–Douglas Utility Function with Two Goods: $x^{1/3}y^{2/3}$



# By the way, this is how Cobb–Douglas Function, $x^{2/3}y^{1/3}$, looks like

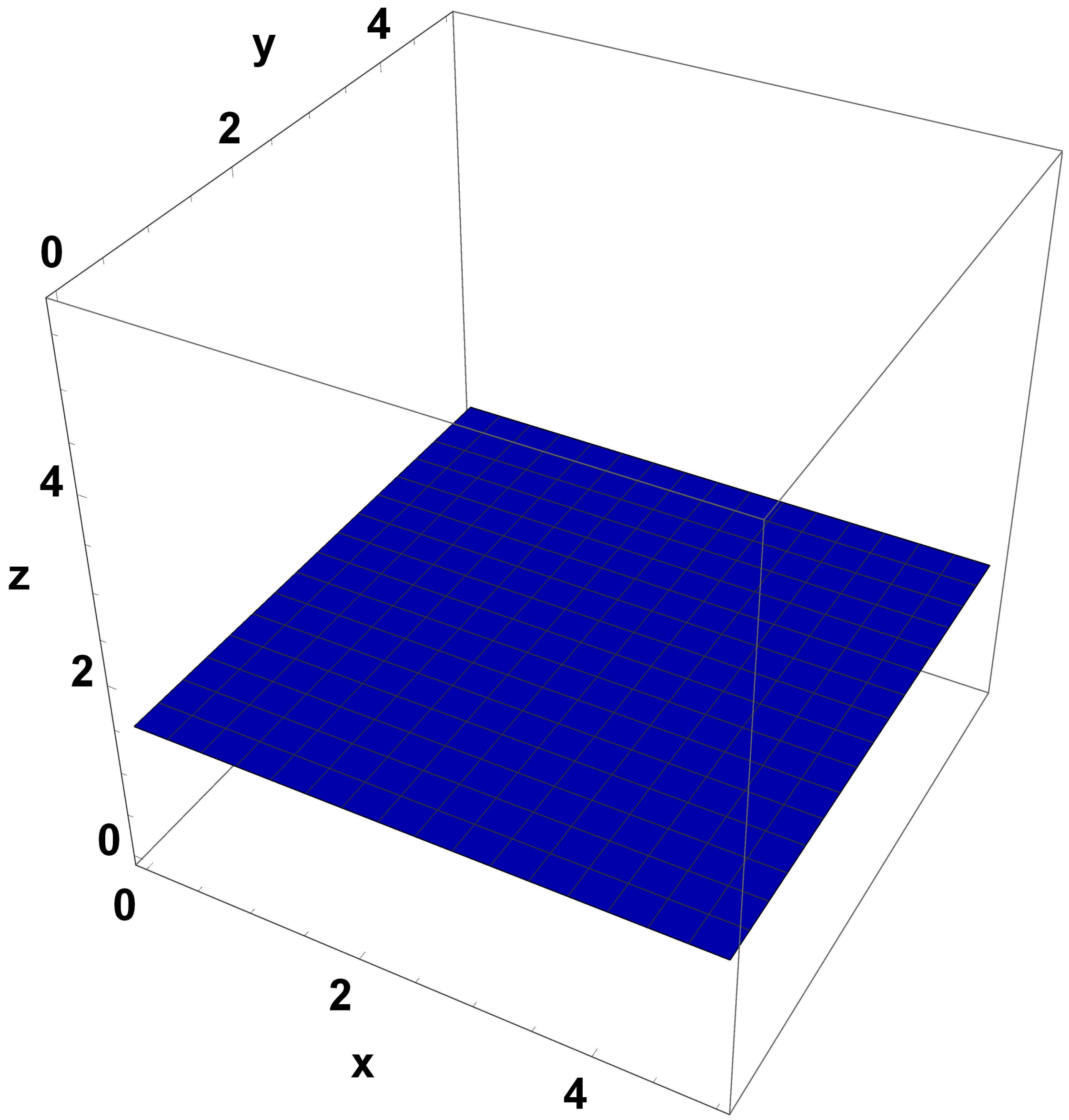


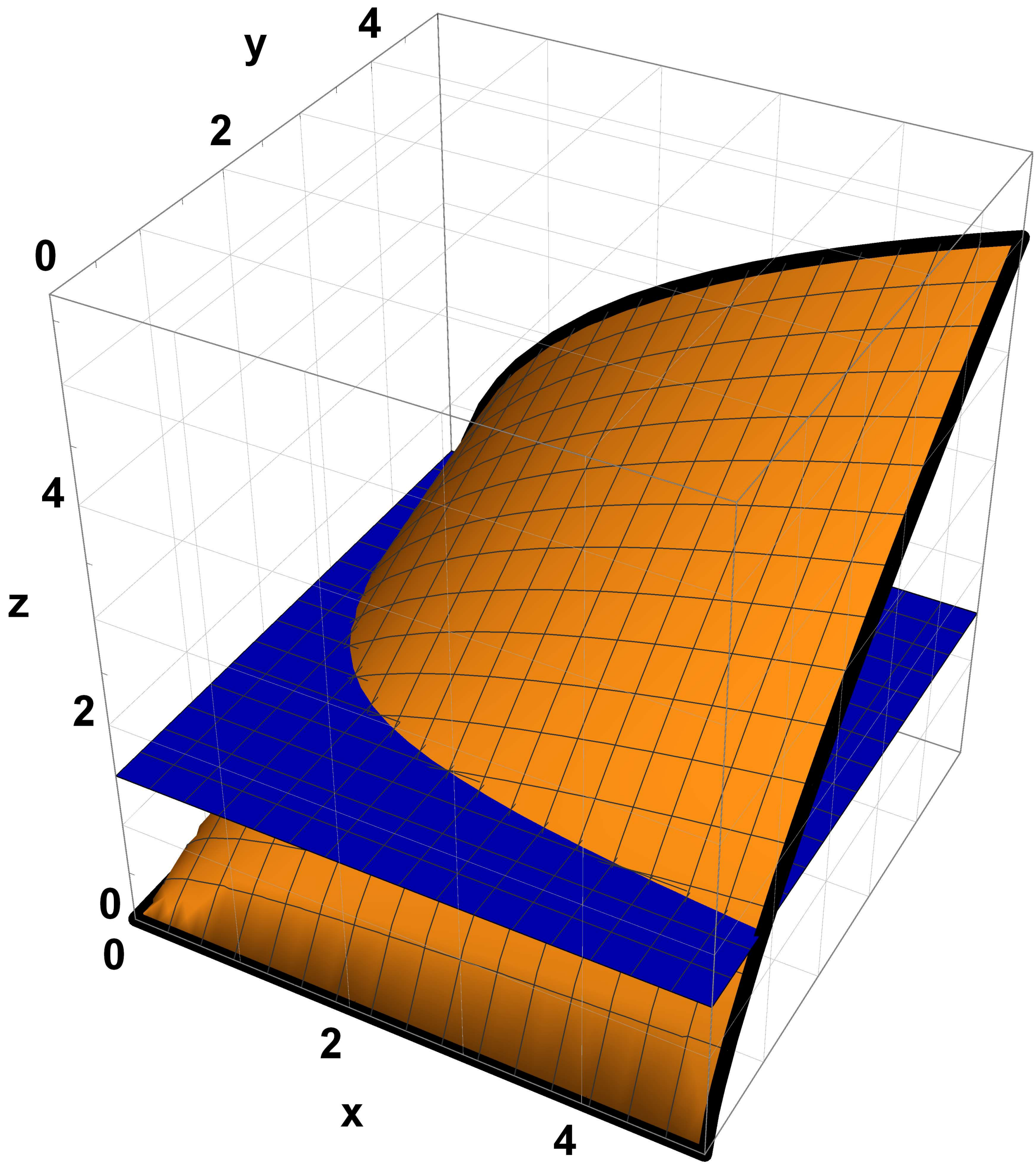

# Here we use the Cobb–Douglas Function: $x^{1/3}y^{2/3}$. Let us see it in a full scale.



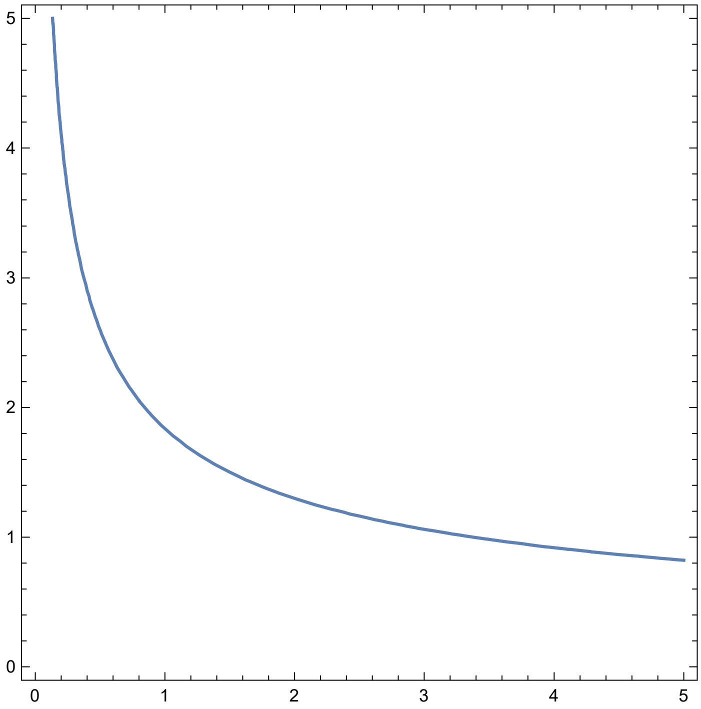

Before thinking about partial differnatiation,
let us consider Indifference Curve

Let us cut z = $x^{1/3}y^{2/3}$ at z = 1.5

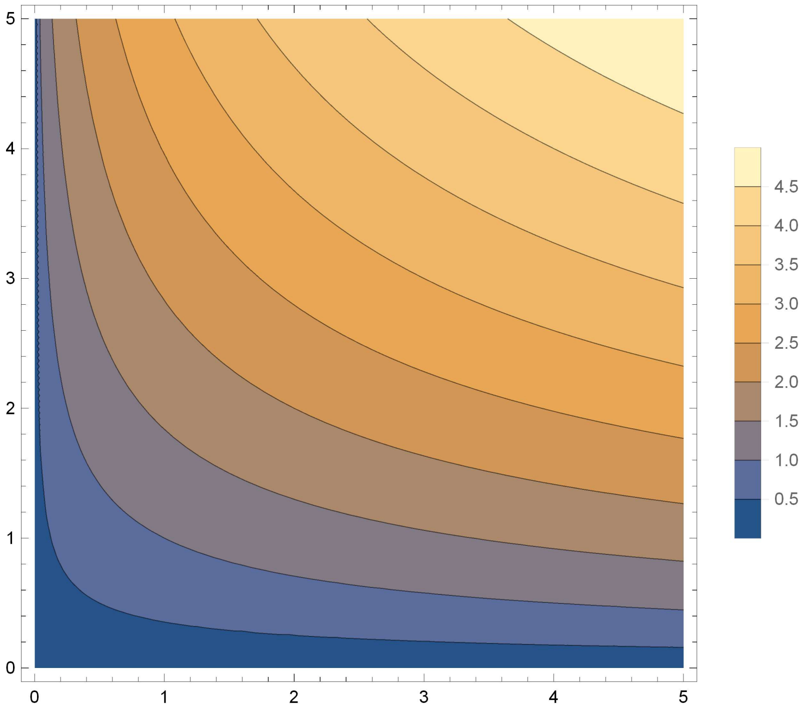

y
4
2
0
4
z
2
0
0
2
x
4




The Contour made by z=1.5
plane gives us an indifference curve.



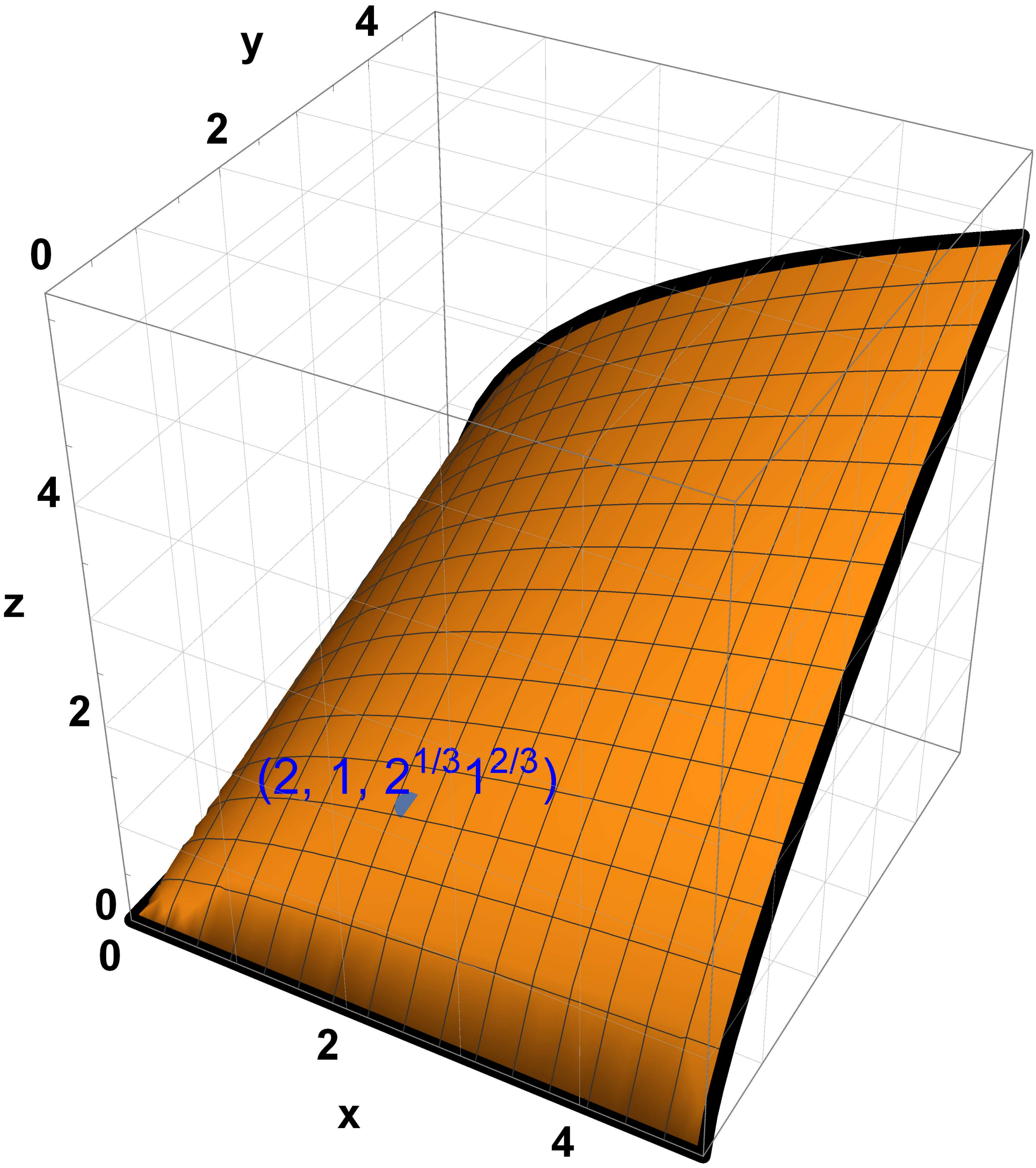

By cutting with different value 'z's, we can have many idifference curves representing different utility level.



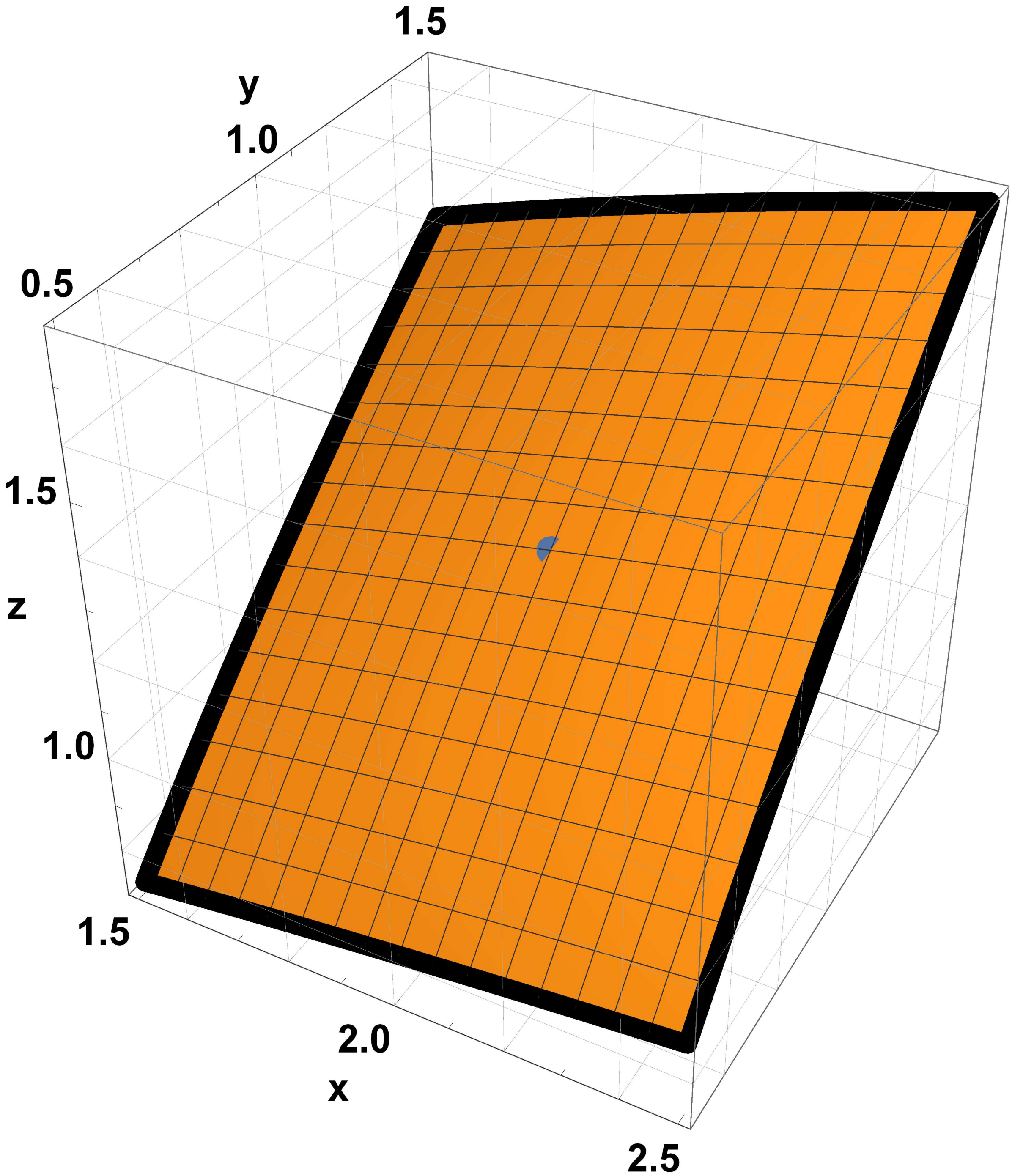

But our main purpose now is to understand partial derivatives!

Recall that a derivative in 2D is a
slope that approximates the original curve
at a specific point. How about in 3D?

Let us consider $x^{1/3}y^{2/3}$ at (x, y) = (2, 1).




Magniy it around (x, y, z) = (2, 1, 2^(1/3) 1^(2/3))

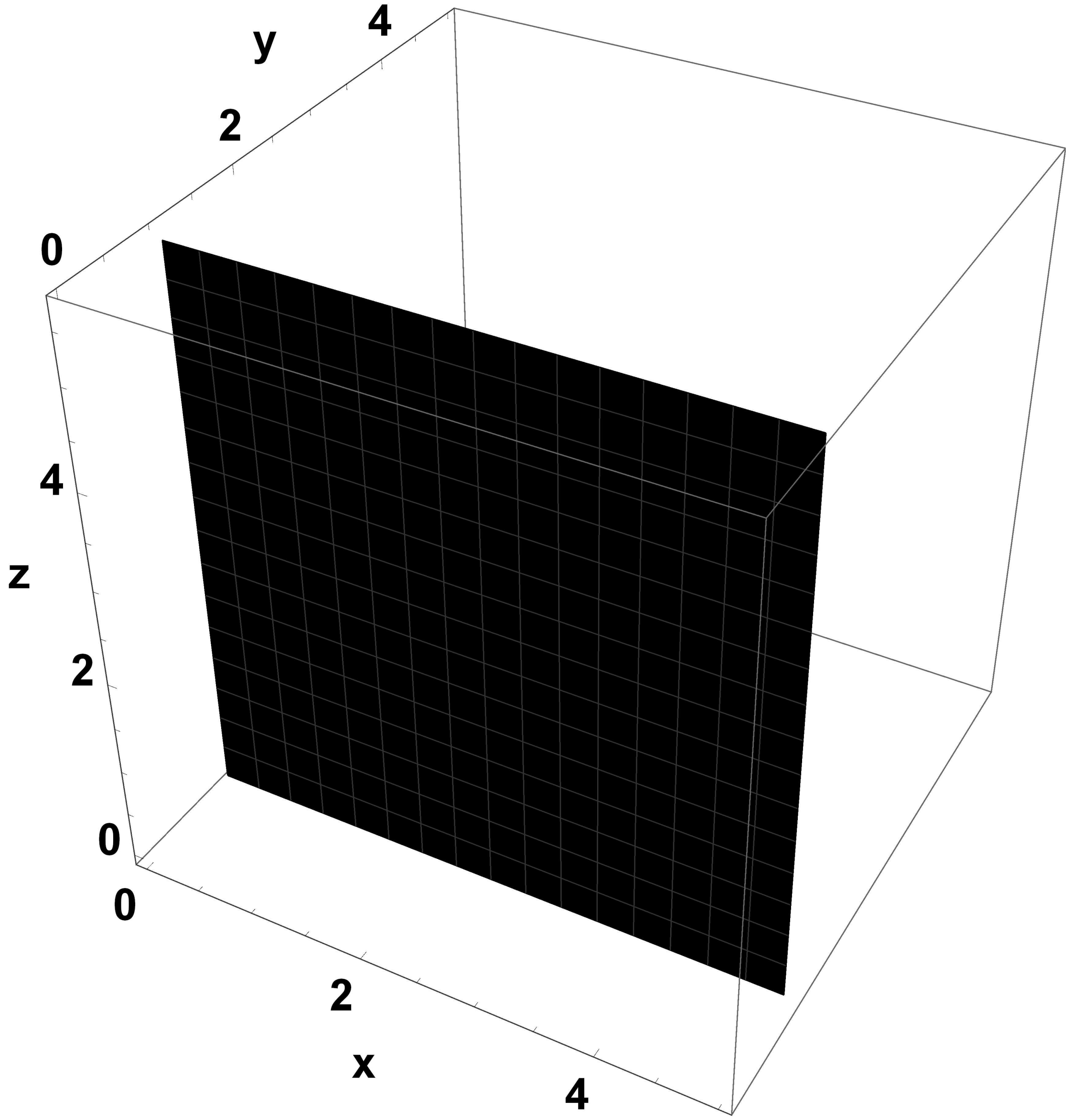


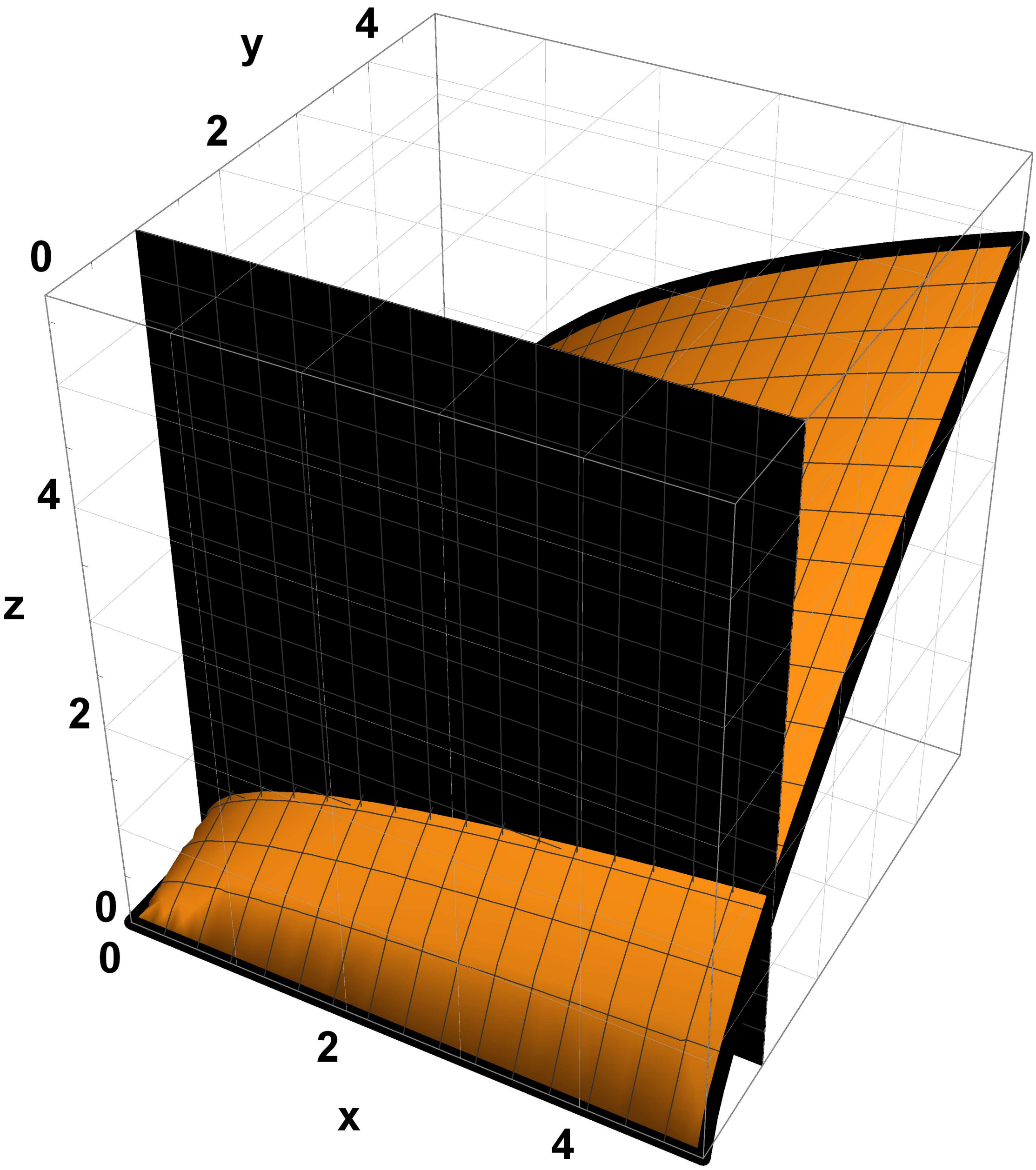

We can guess that a plane is a strong candidate for approximating this curved surface at (x, y, z) = (2, 1, 2^(1/3)*1^(2/3))

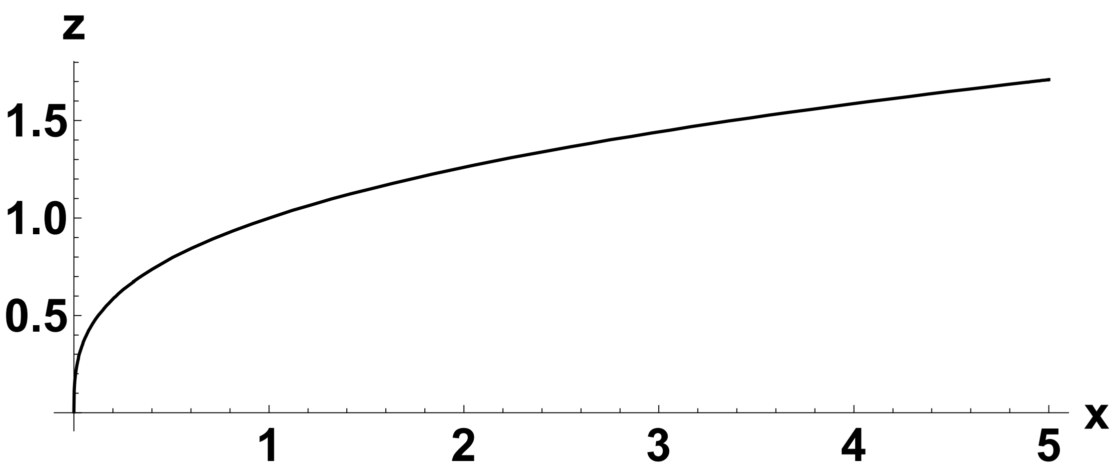
# Let us confirm our guess.

Fix y at some value, then look at our 3D graphic from x axis. The 3D becones like a 2D.

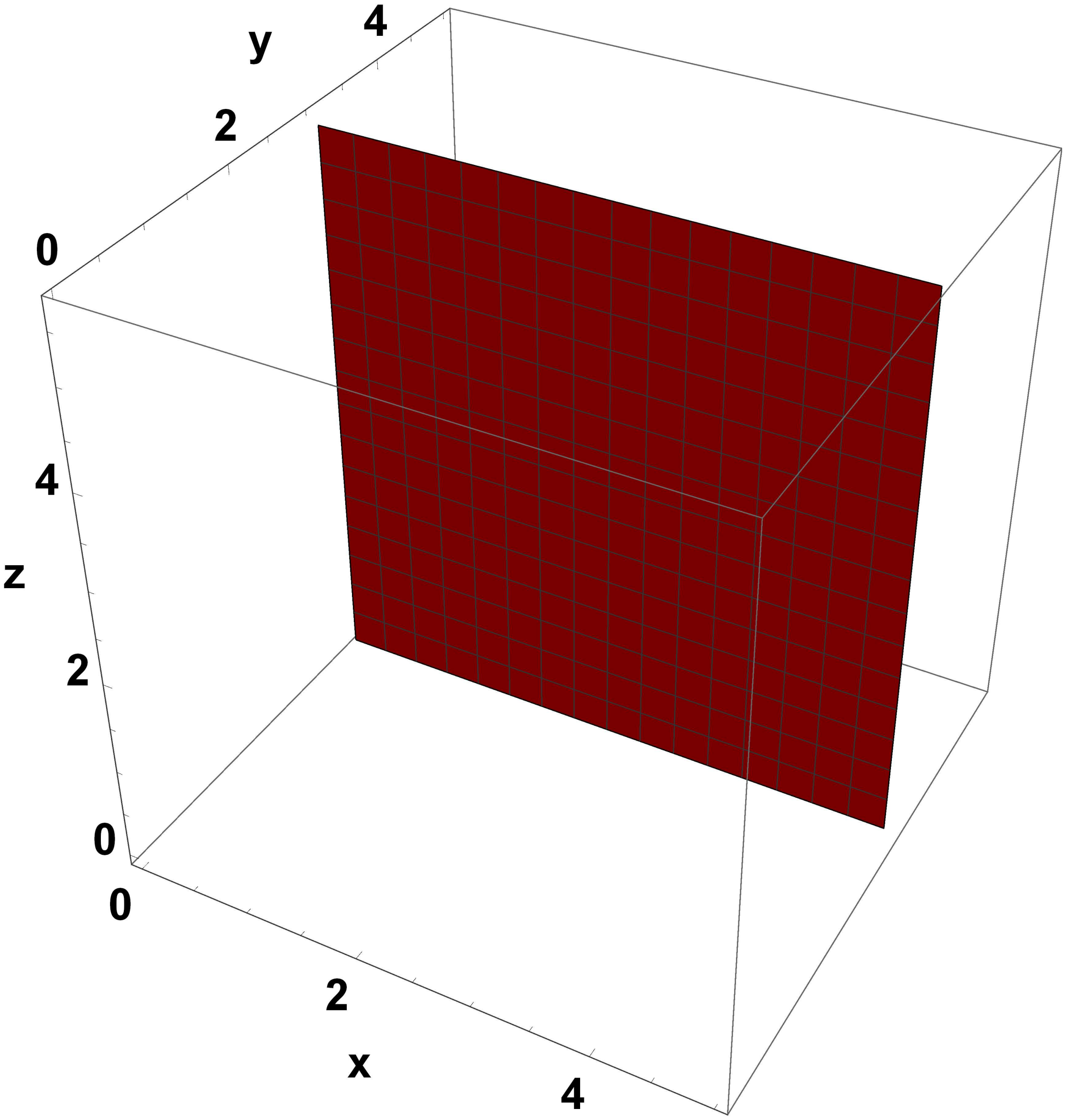
Here we fix y at 1.

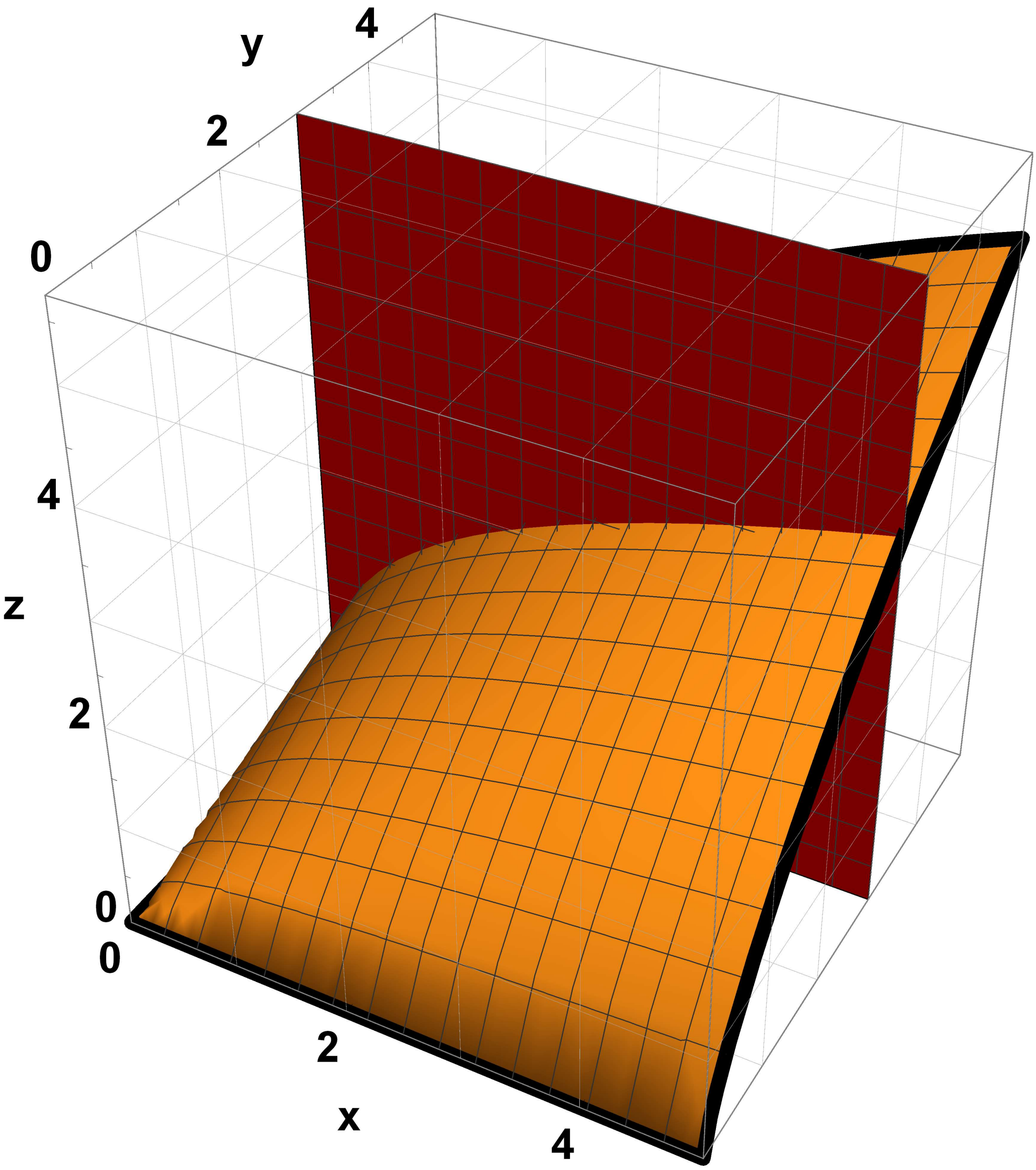



y
4
2
0
4
z
2
0
0
2
x
4

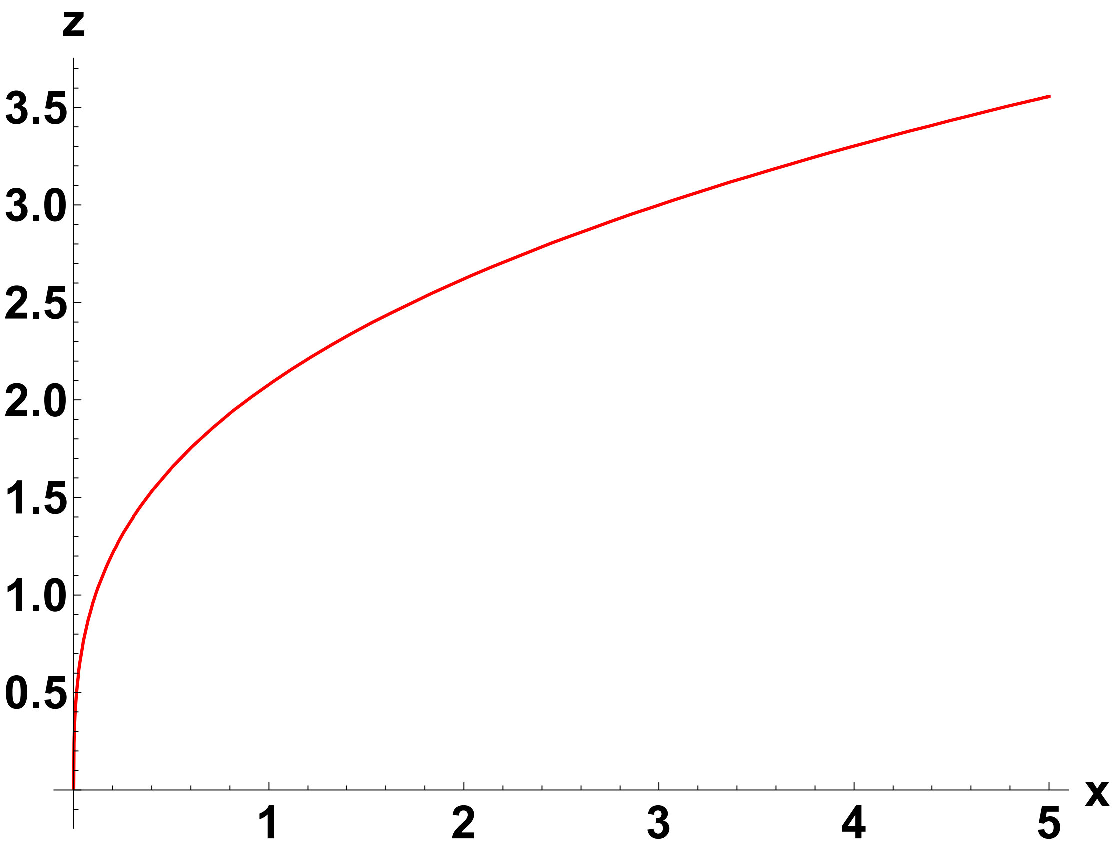

z
1.5
1.0
0.5
1
2
3
4
5
x

If we change the value of y, the pseudo 2D graphs change its shape a bit. Here we fix y at 3.

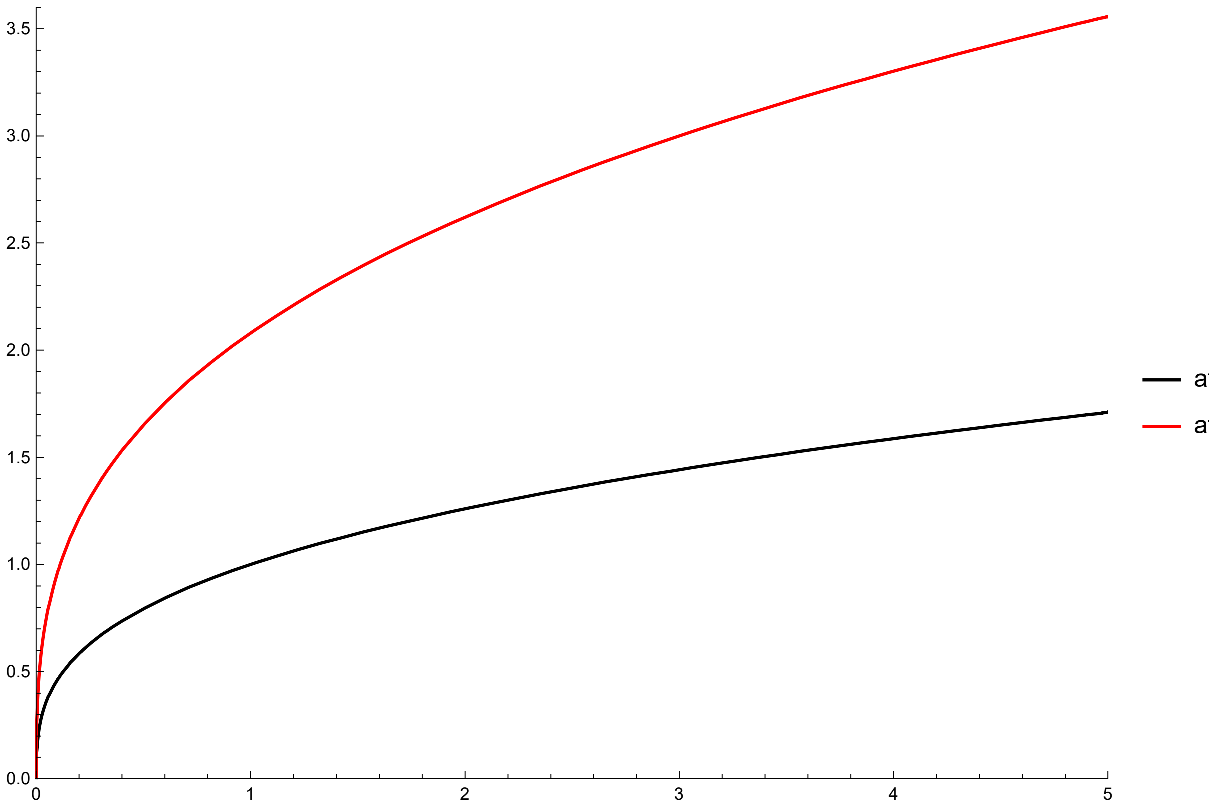





The contour is another 2D graphic




We can see the differnces by the cutting values of y



# Now we focus on the 2D with y = 1.

Differentiate (x^(1/3))*(1^(2/3)) with respect x,

$$\frac{1}{3\,x^{2/3}}$$

Differentiate x^(1/3))*(y^(2/3) with respect x. Here, we are doing partial differnetiation. Please accept the result at this stage.

$$\frac{y^{2/3}}{3\,x^{2/3}}$$

Derivative (= scalar) of 2D at x = 2

0.209987

Partial derivative of 3D at (x, y) = (2, 1)

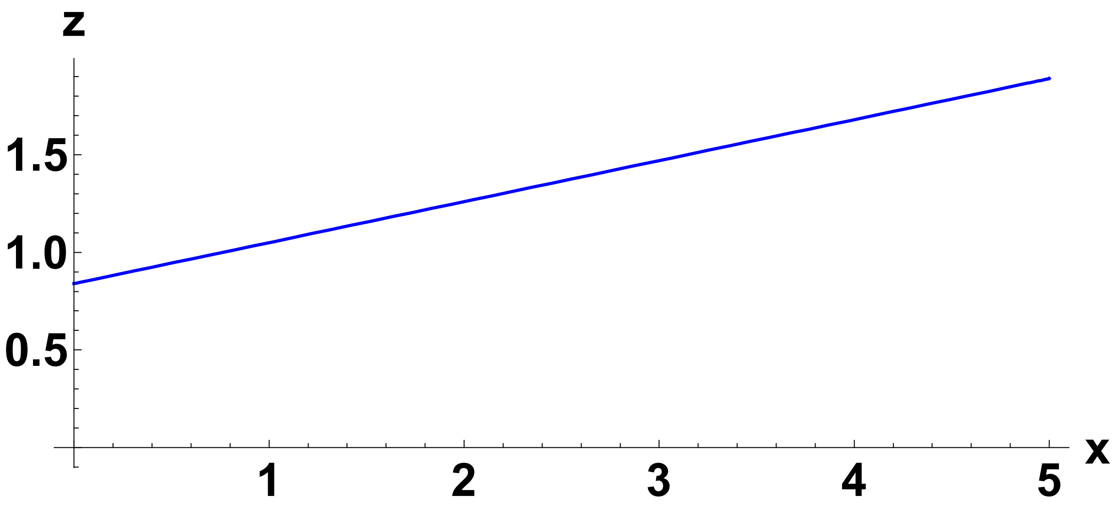
0.209987

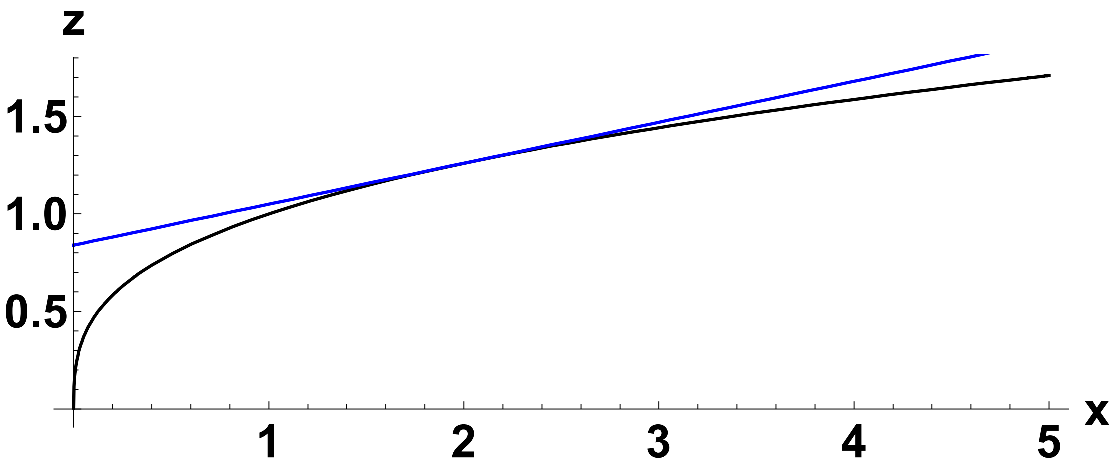
In short, partial derivative in 3D is a `slope' of 2D after fixing y (or x)

Partial derivative with respect to x at (x, y, z)=(2, 1, 2^(1/3))*(1^(2/3)) is a slope of (x^(1/3))*(1^(2/3)) at x = 2

With the 'scalar value' of slope and the information that the slope is evaluated at (x, z) = (2, 2^(1/3))*(1^(2/3)), we can derive a tangent line

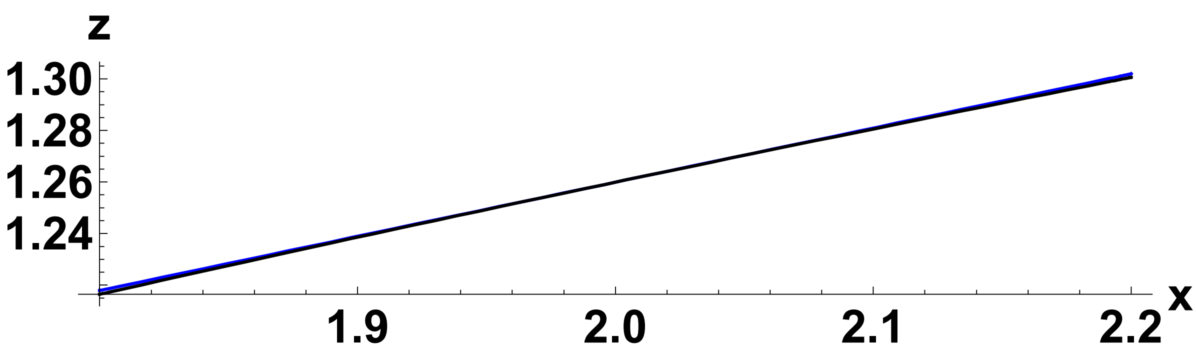
The derived tangent line is z = 0.8399473665965822 + 0.20998684164914552*x



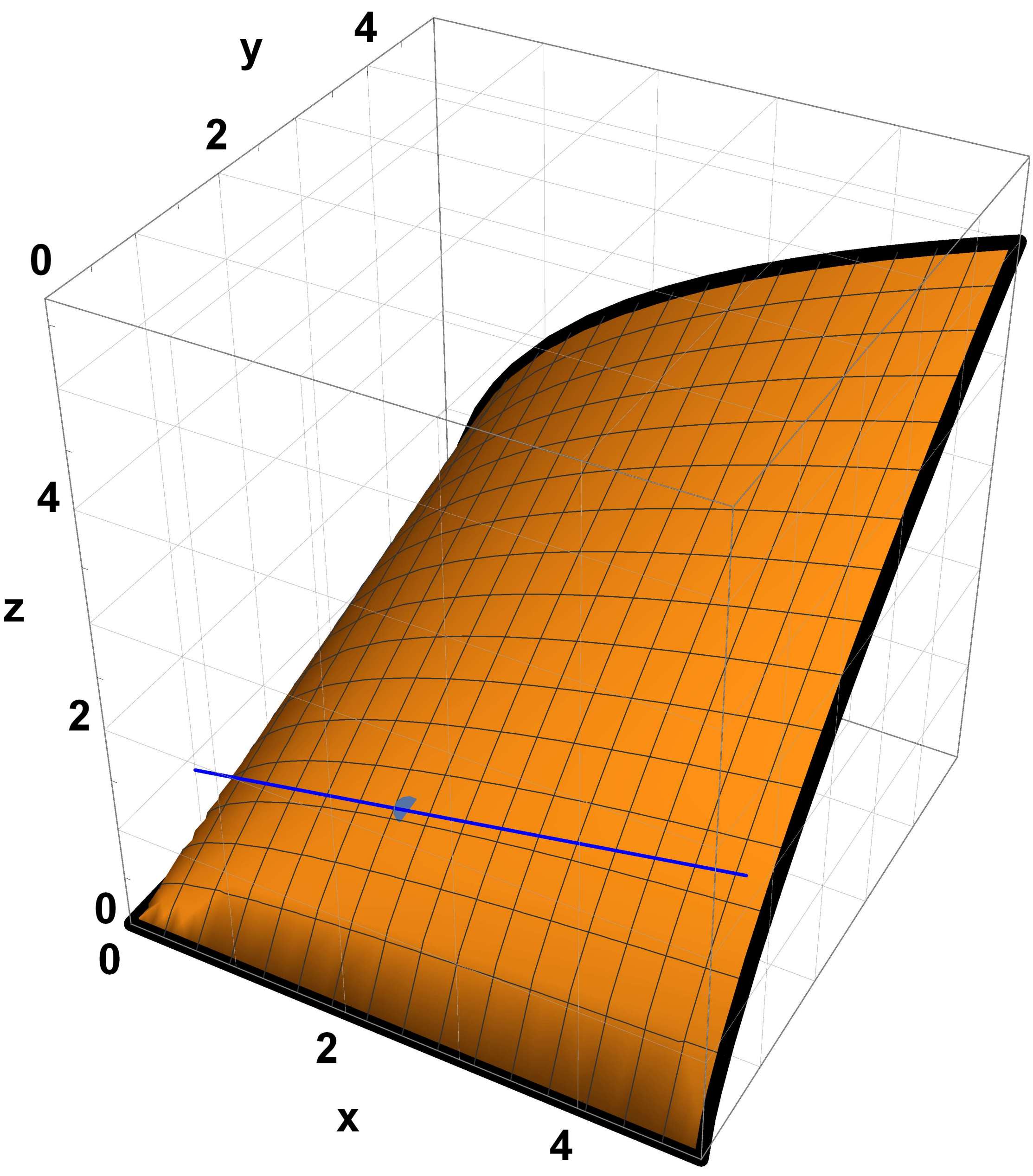




As usual, tangent lines derived from derivatives
are Very good approximates of curves

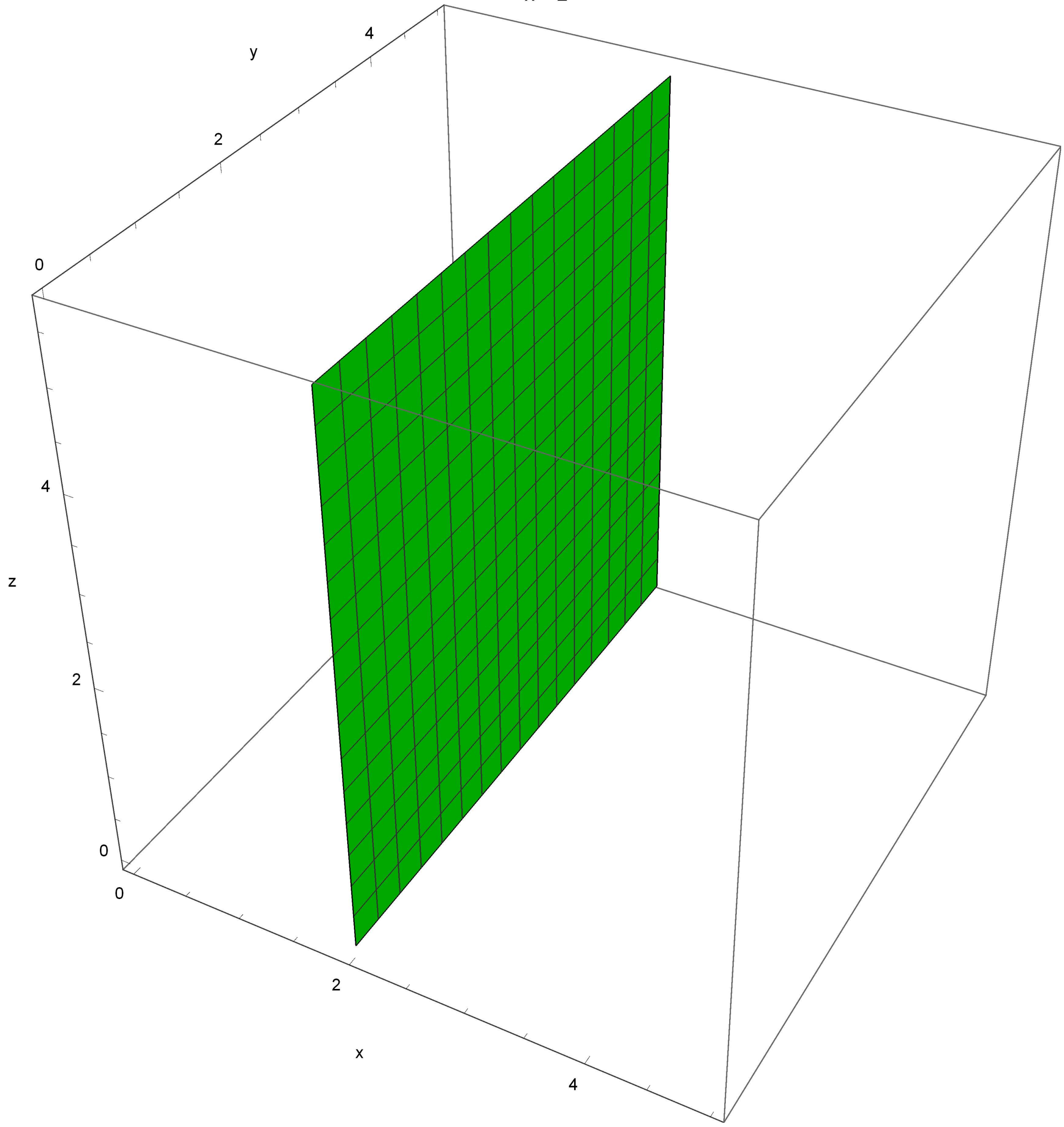
Let us magnify the 2D graph
and the tangebt line around x = 2

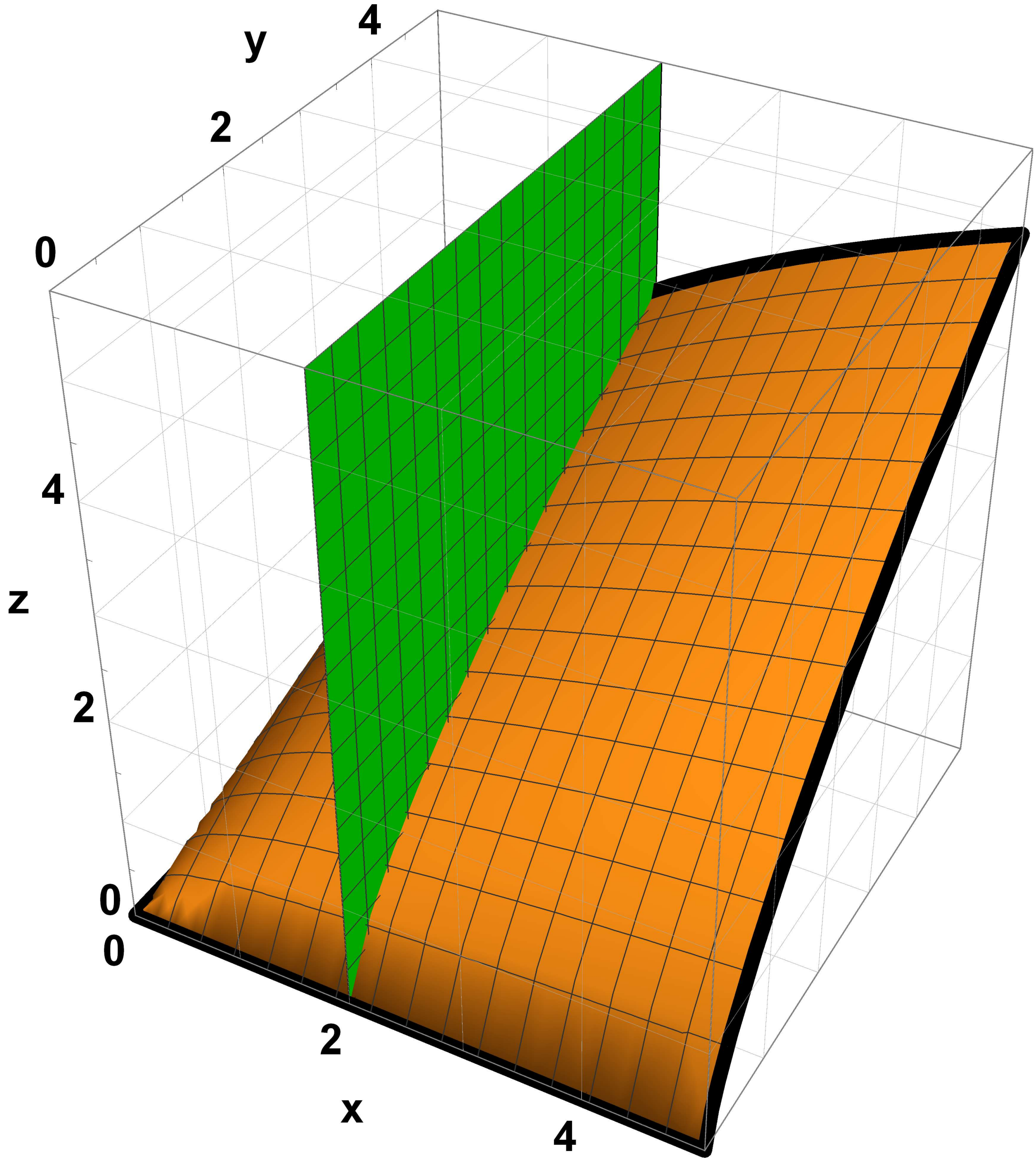



Let us have a look at partial
derivative with respect to x at (2,1) on 3D

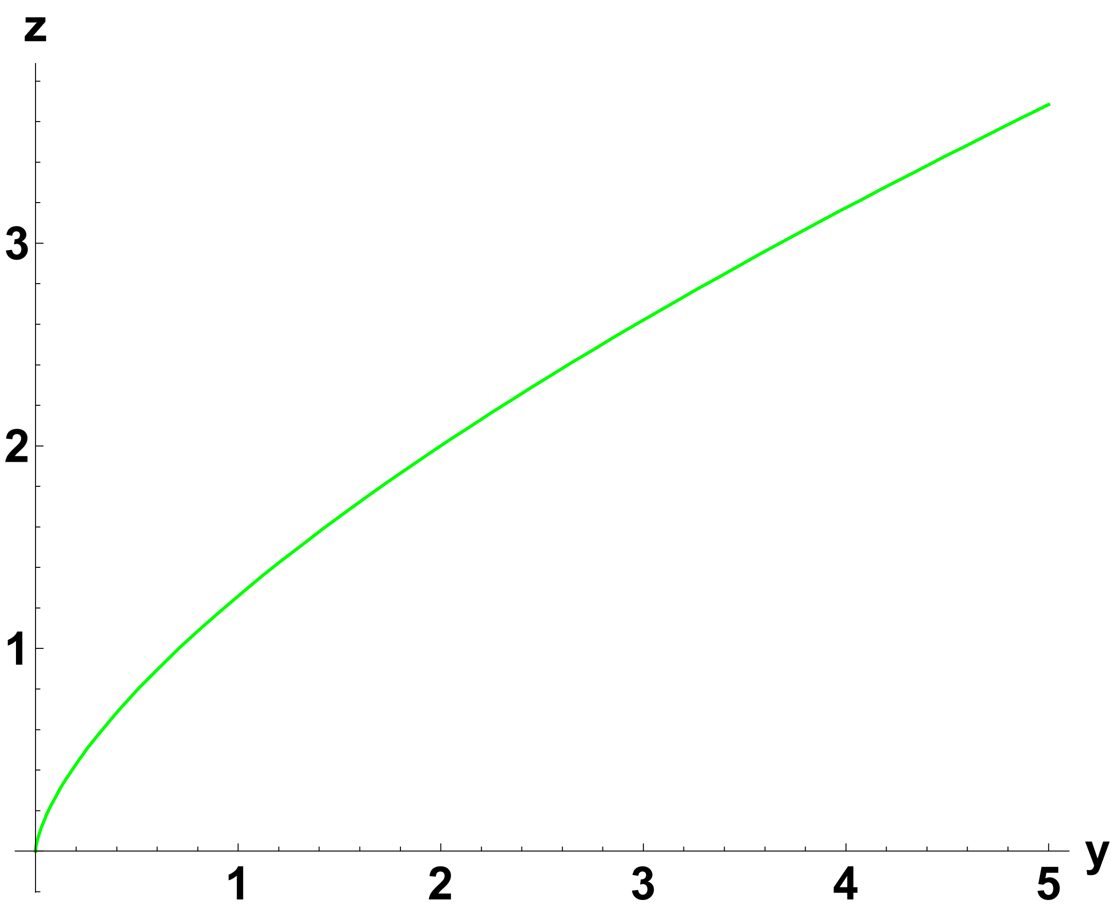



We can do the similar procedure by fixing x at 2 x = 2






The contour is a 2D graphic




Differentiate (2^(1/3))*(y^(2/3)) with respect y,

$$\frac{2 \times 2^{1/3}}{3\, y^{1/3}}$$

Differentiate (x^(1/3))*(y^(2/3)) with respect y. Here, we are doing partial differnetiation.

$$\frac{2\, x^{1/3}}{3\, y^{1/3}}$$

Derivative of 2D at y = 1

0.839947

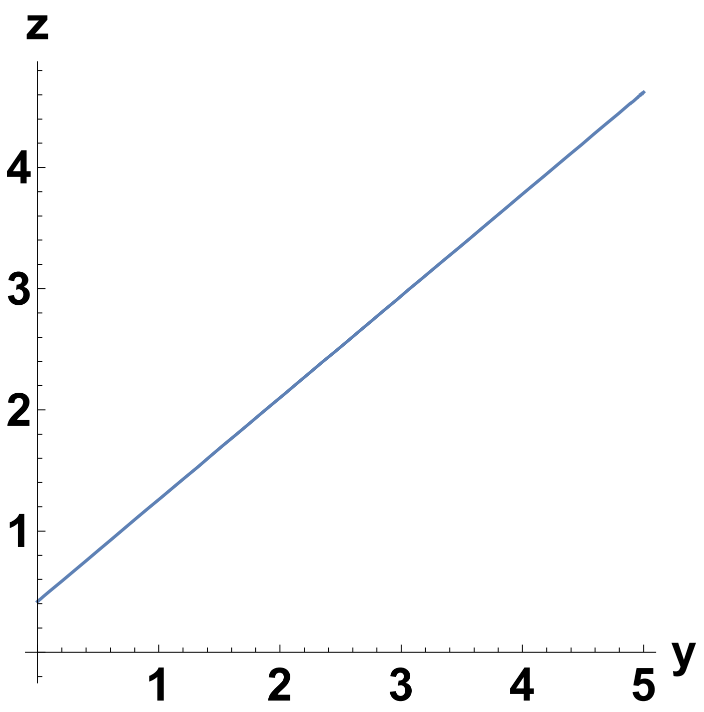
Partial derivative of 3D at (x, y) = (2, 1)

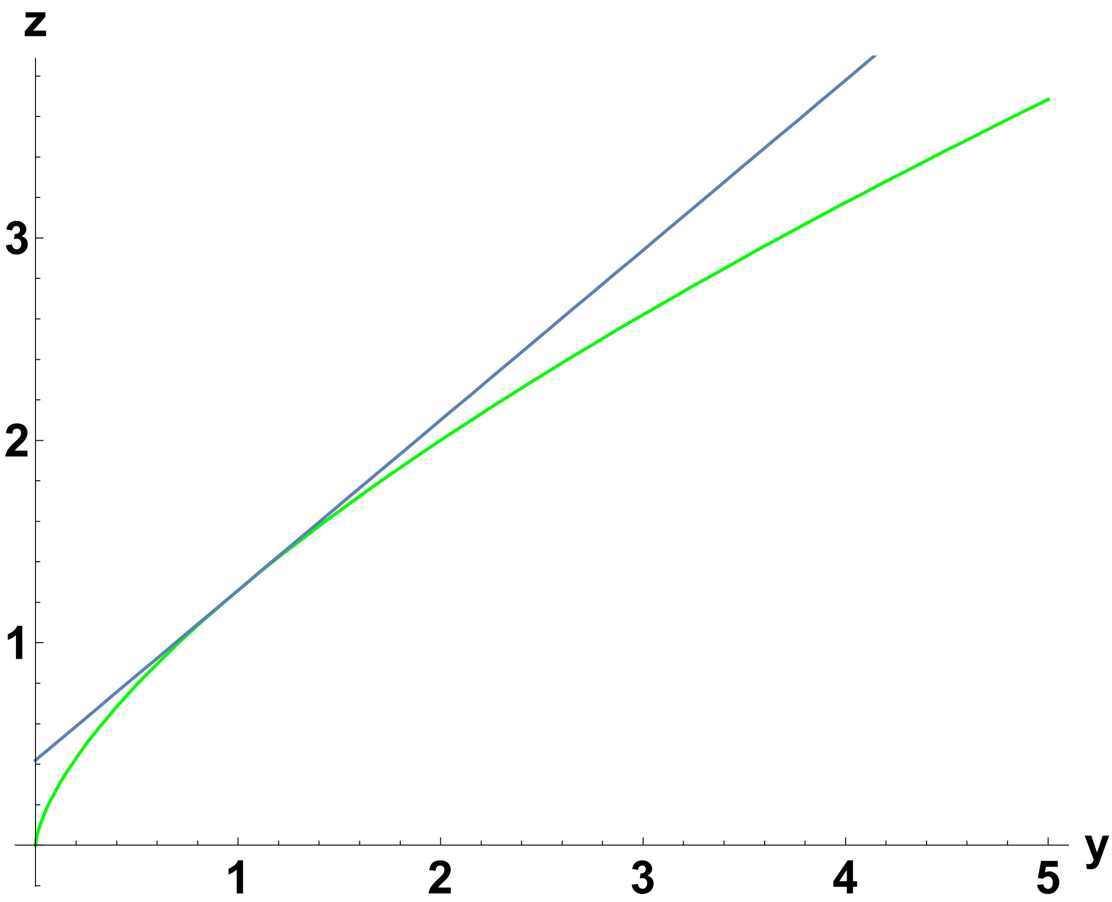
0.839947

Partial derivative with respect to y at (x, y, z)=(2, 1, 2^(1/3))*(1^(2/3)) is a slope of (2^(1/3))*(y^(2/3)) at y = 1

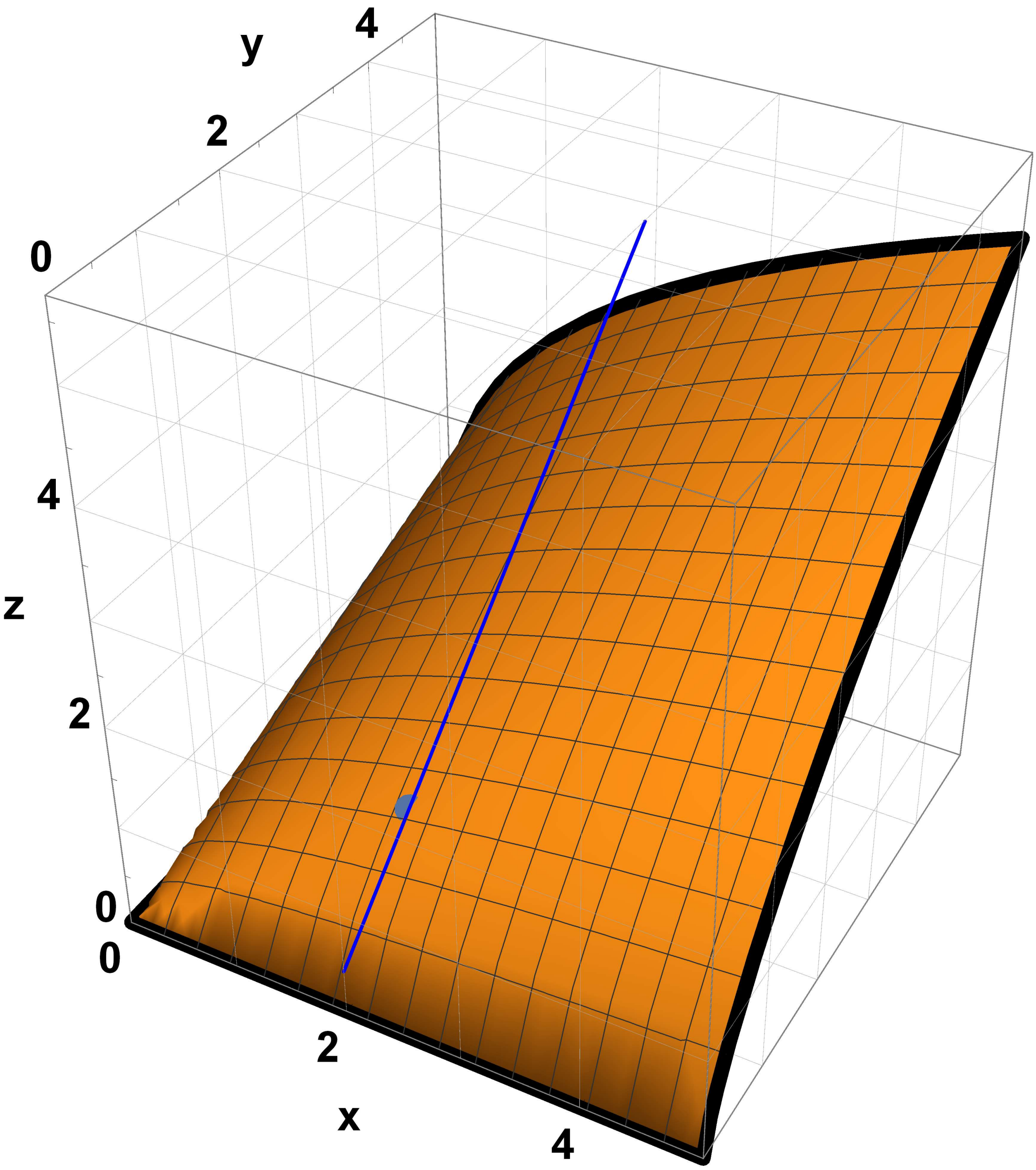
With the 'scalar value' of slope and the information that the slope is evaluated at (y, z) = (1, 2^(1/3))*(1^(2/3)), we can derive a tangent line

The derived tangent line is z = 0.4199736832982911 + 0.8399473665965821*y

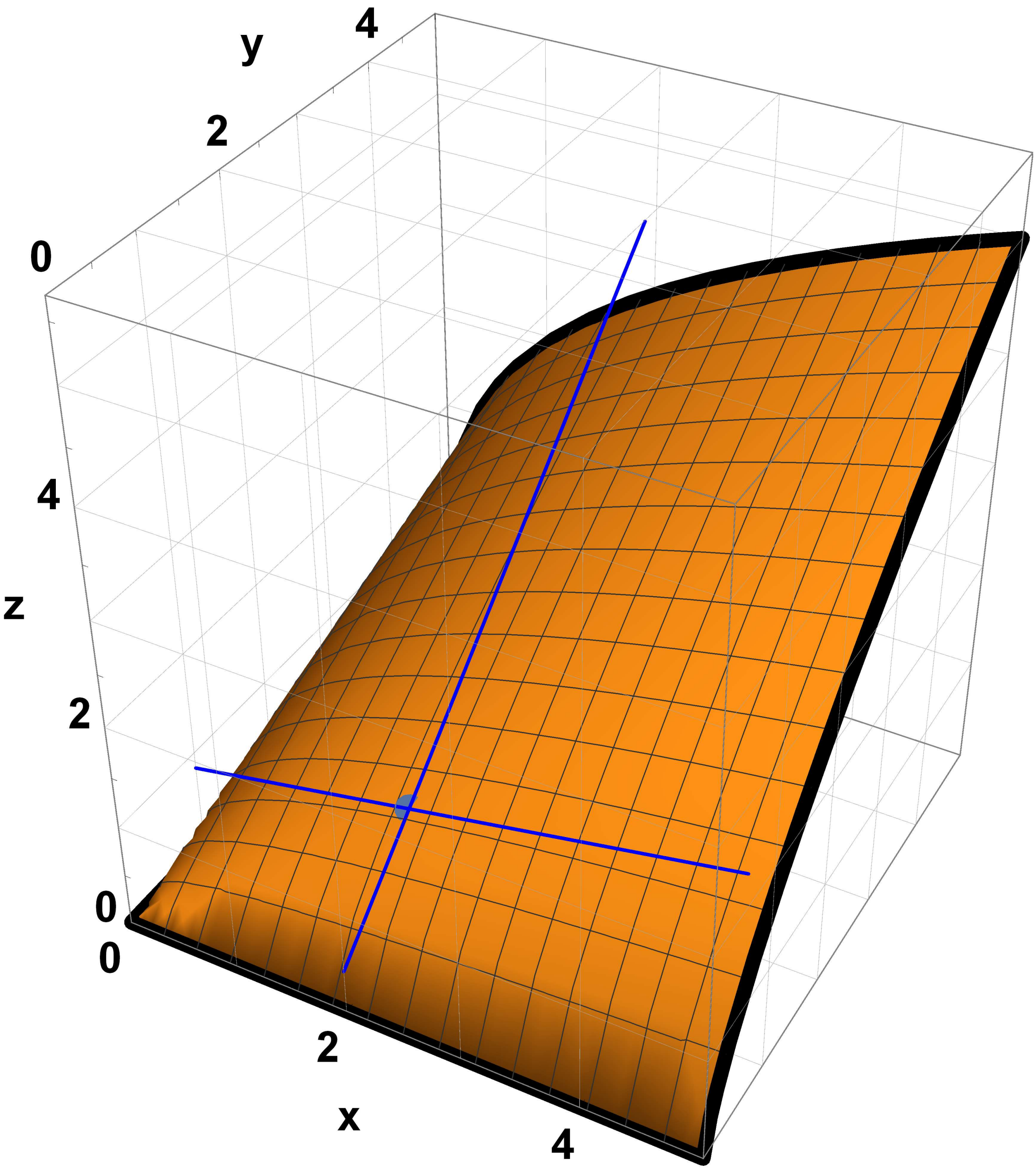





Partial derivative with respect to y at $(2, 1, 2^{1/3}1^{2/3})$ on 3D



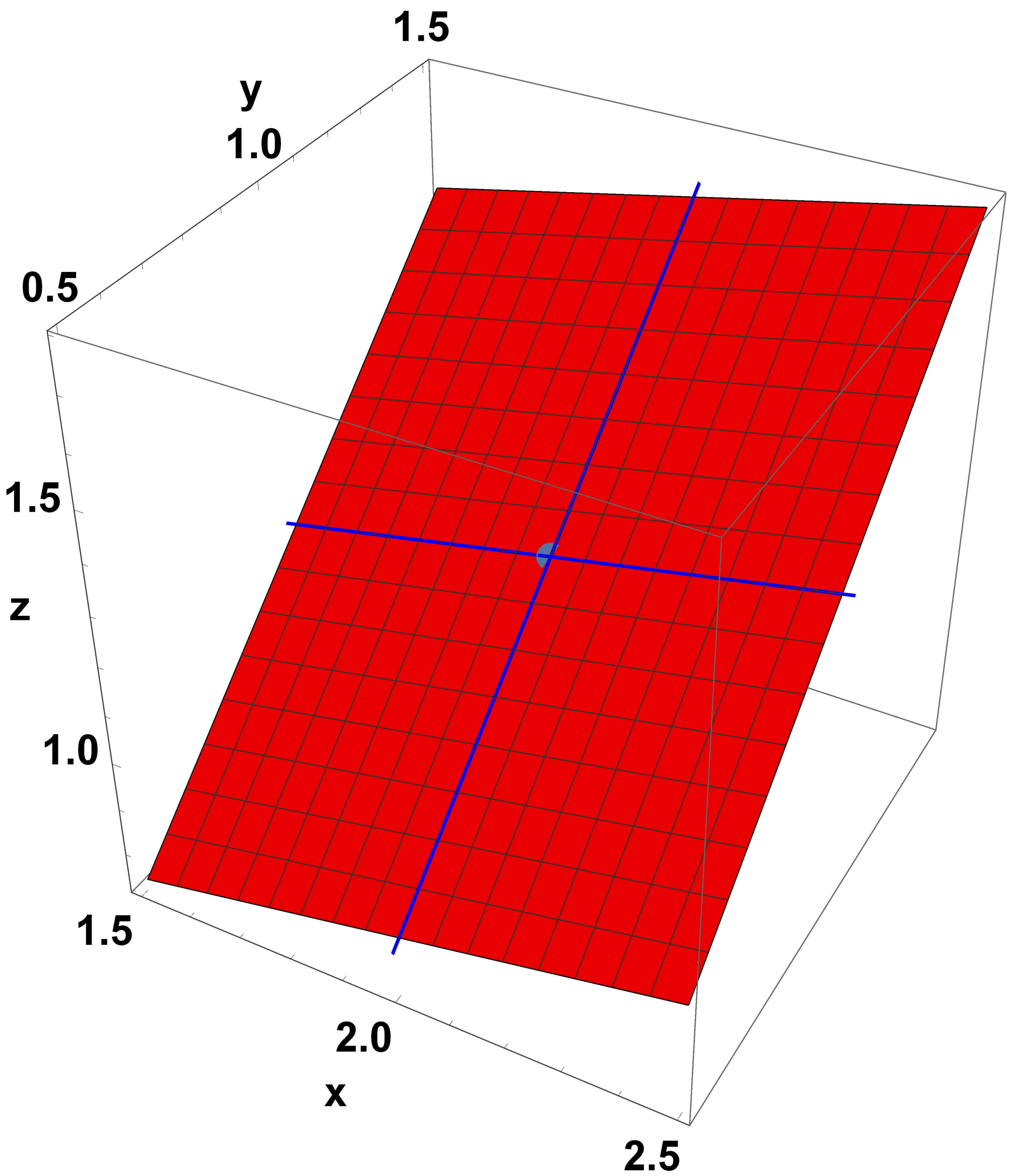

If we combine the two tangent lines on 3D




The two tangent lines in 3D give us a plane:

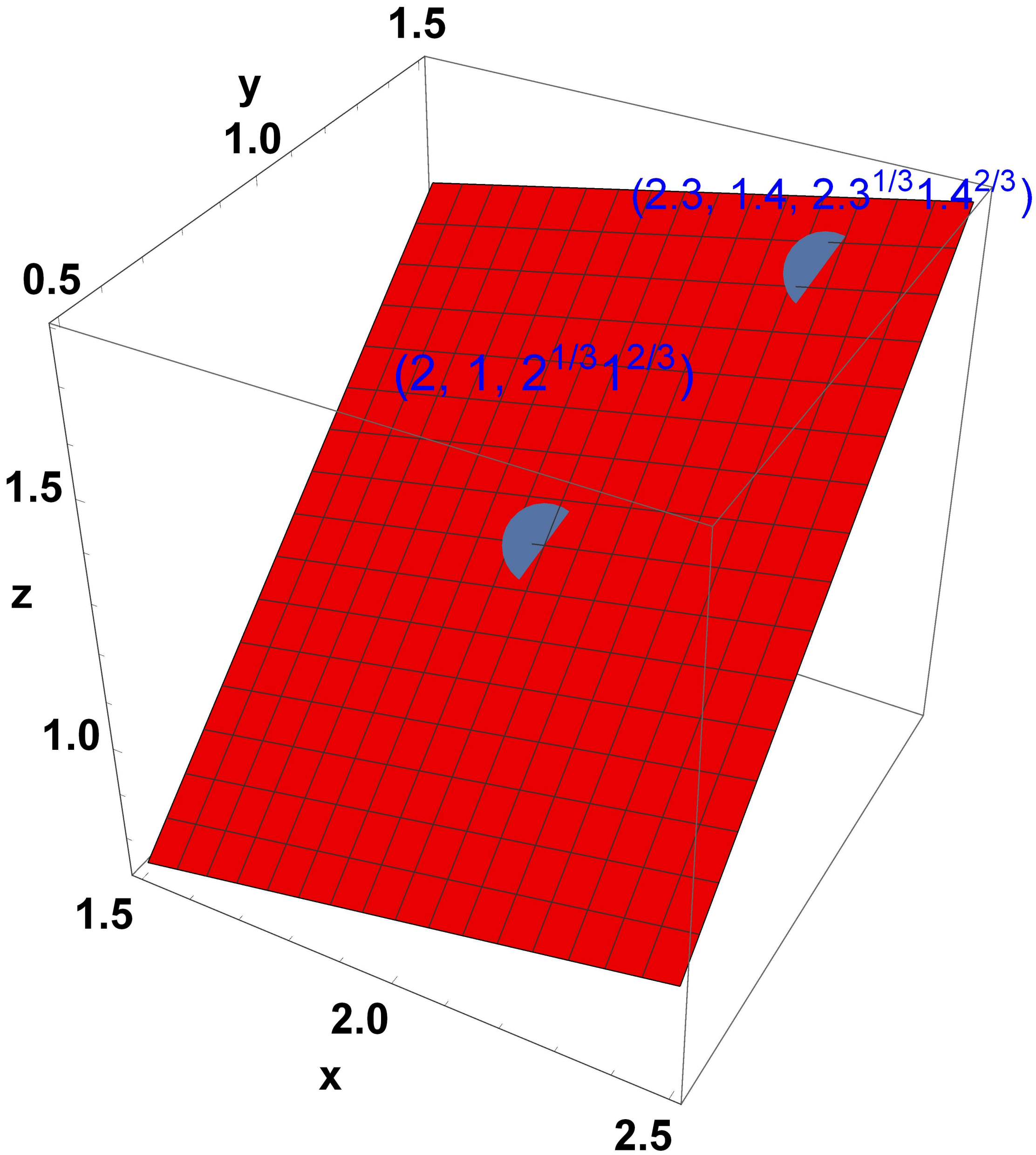
A Tangent Plane on (x, y, z) = (2, 1, $2^{1/3}1^{2/3}$)

$$\frac{x}{3\times 2^{2/3}}+\frac{2}{3}\times 2^{1/3}\,y$$



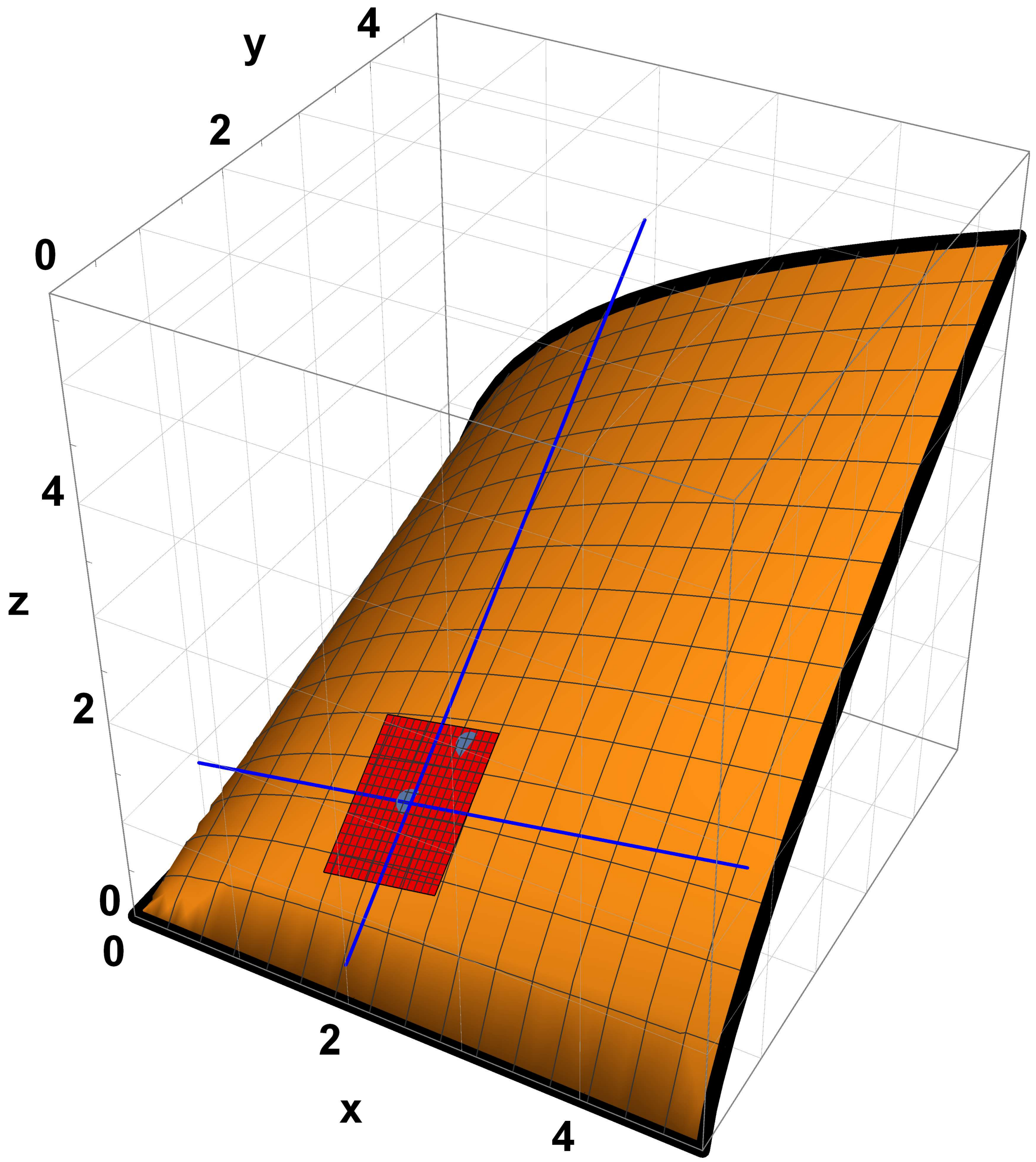

Recall that our goal is to use derivatices (calculus) in economics: linear approximation of 'non–linear' relationships

Can the tangent Plane at (x, y, z) = $(2, 1, 2^{1/3}1^{2/3})$ approxiapiate the curved surface at, for example, (x, y, z) = $(2.3, 1.4, 2.3^{1/3}1.4^{2/3})$ well?

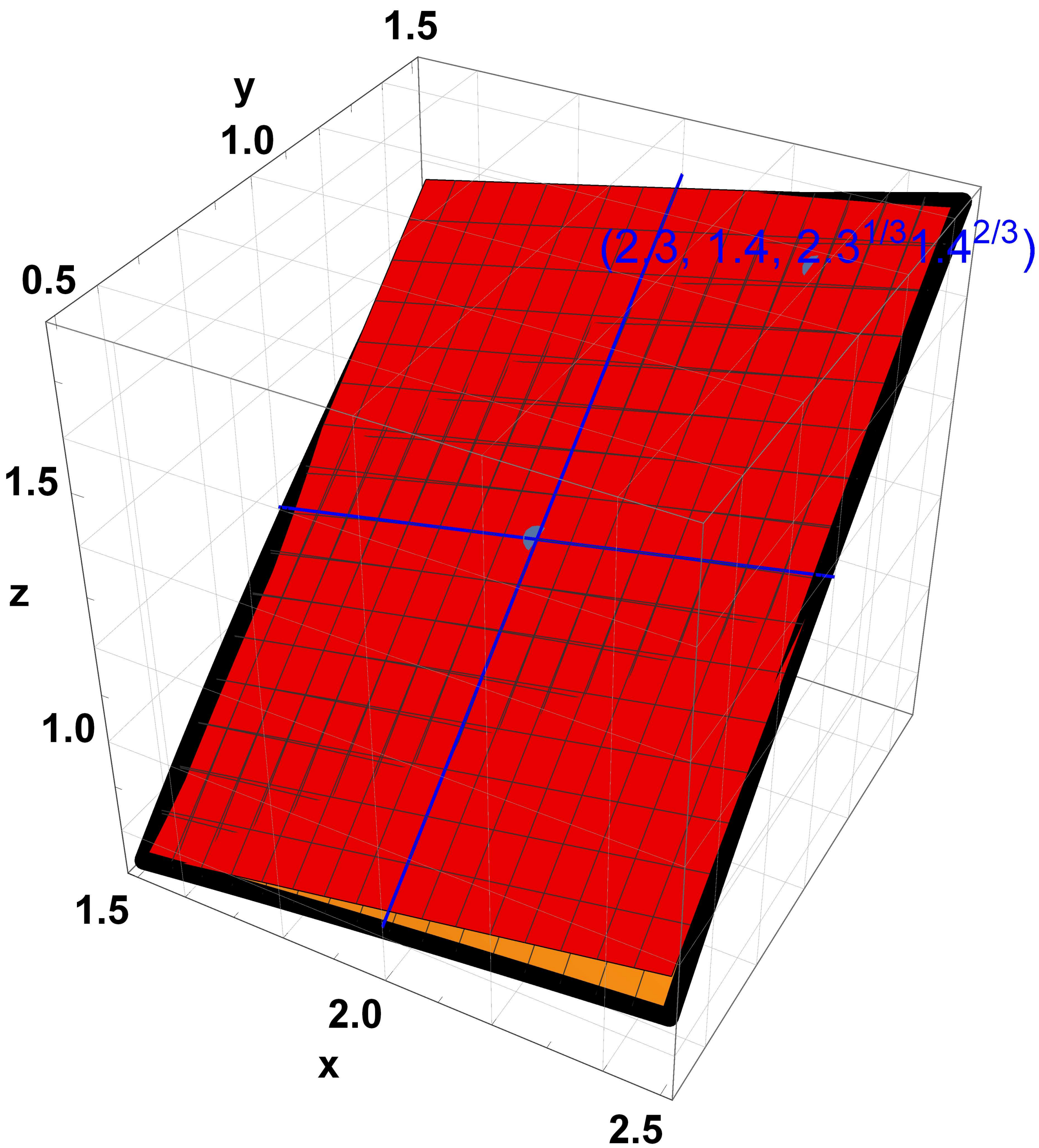



You can see that the point at (x, y, z) = (2.3, 1.4, $2.3^{1/3}1.4^{2/3}$) dipped into the plane a little bit, but the plane is surely a good approximate.

Let us see this approximation by the plane on the oroginal 3D surface of Cobb–Douglas function




Looks like a good approximation, but difficult to see. Let us magnify the graph around (x, y, z) = (2, 1, $2^{1/3}1^{2/3}$)




With this 3D Graph, we can easily understand so−called 'Total Differentiation'

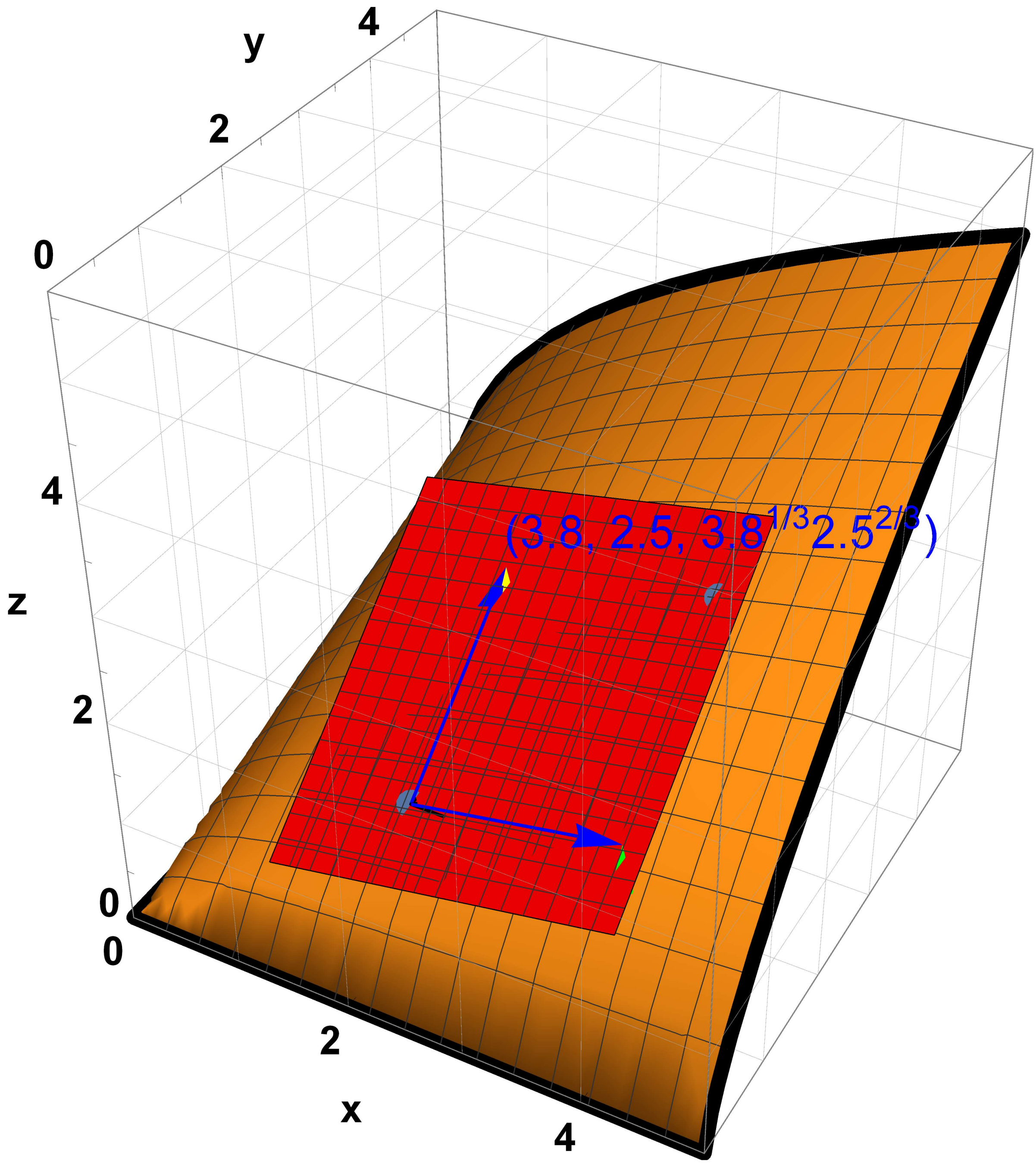
Let us rewrite our Cobb−Douglas function in an abstract way: $z = f(x, y) = x^{1/3}y^{2/3}$

'Total Differentiation': $dz = \frac{\partial}{\partial x}f(x, y)dx + \frac{\partial}{\partial y}f(x, y)dy.$

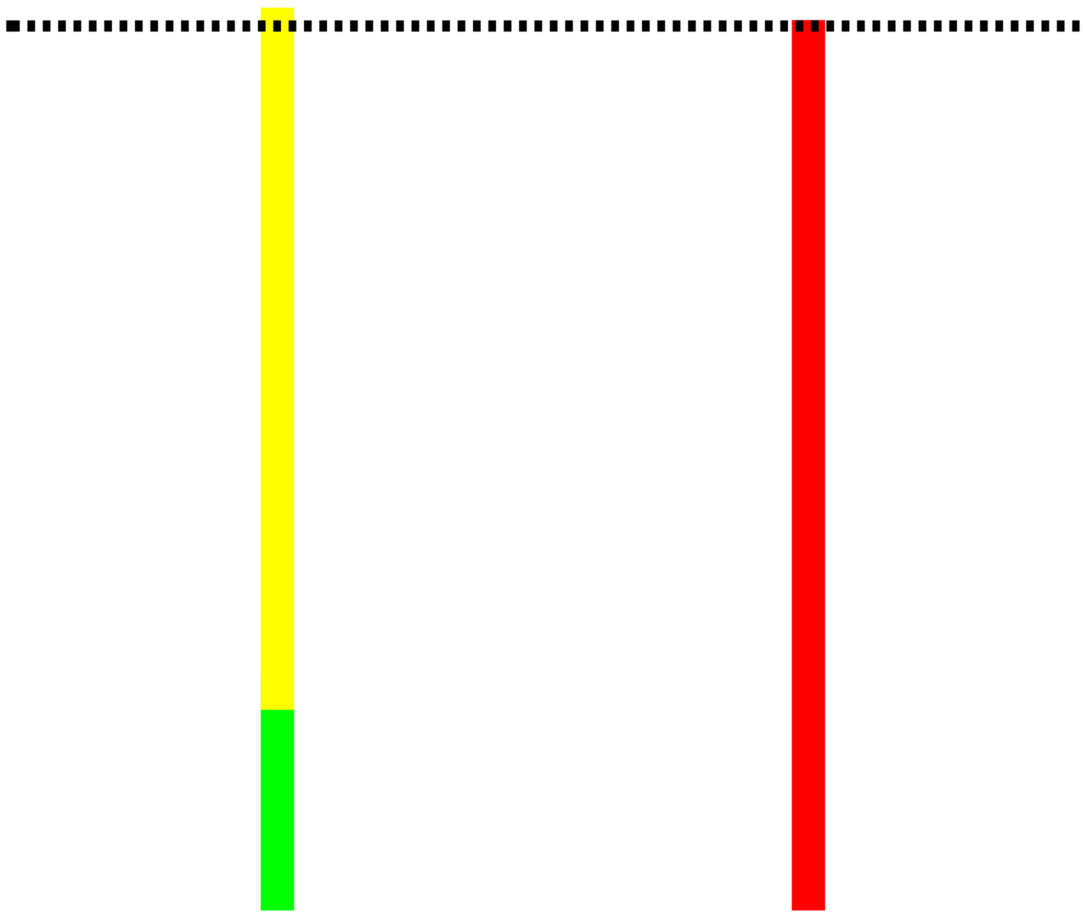
In plain English, we would like to know
how much does the functional value z
change when both x and y change a little bit

To understand the total differnatiation on a
graph, let us consider fairly big changes in x
and y: $dx = 1.8 = (3.8-2)$, $dy = 1.5 = (2.5 -1)$

See the at Graph at first.




What total differentiation does is:



The tangent plane is avobe
the curve => A little bit overestimation:

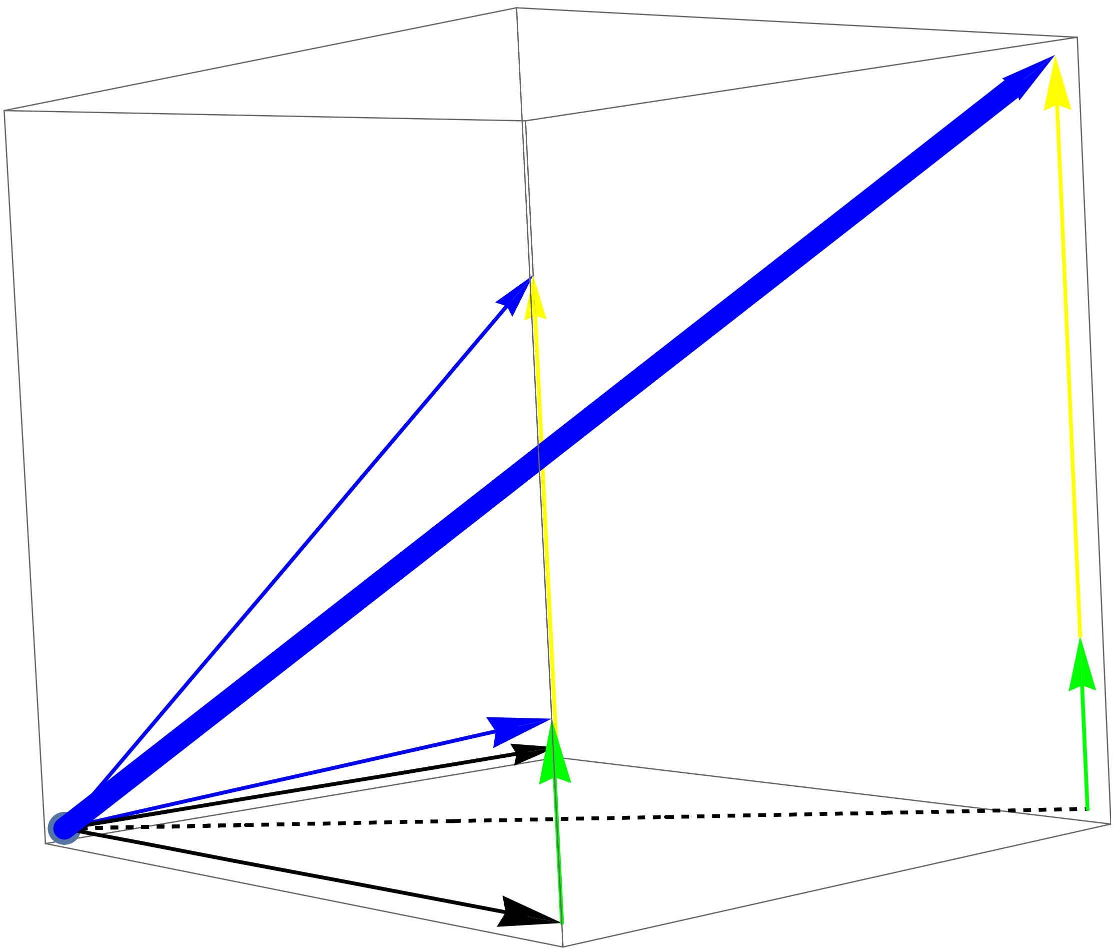
The length of red line is $3.8^{1/3}2.5^{2/3} - 2^{1/3}1^{2/3}$ =

1.61453

while the sum of green and yellow is

1.6379

An alternative and more intuitive interpretation is to think a total differntiation as a summation of 2 vectors



Here the thick blue arrow is the gradient vector: $\nabla f = (f_1, f_2)$ at $(x, y) = (2, 1)$. It gives us the steppest increase (slope) at $(2, 1)$.